CALCULUS
Continuity
NEW
0210-1. a. At which numbers is the function $f$, shown above, discontinuous?

b. For each of the numbers, given in Part a, where $f$ is discontinuous, state whether or not $f$ is continuous from the LEFT at that number.

c. For each of the numbers, given in Part a, where $f$ is discontinuous, state whether or not $f$ is continuous from the RIGHT at that number.
Display the graph of a function $f$

s.t. \[ \lim_{x \to -2^-} f(x) = -1, \quad \lim_{x \to -2^+} f(x) = 2, \]

and s.t. \[ f(-2) = 2, \]

and s.t. \[ \lim_{x \to 1} f(x) = 3, \quad f(1) = 2, \]

and s.t. \[ \lim_{x \to 2^-} f(x) = \infty, \quad \lim_{x \to 2^+} f(x) = -\infty, \]

and s.t. \[ \lim_{x \to -\infty} f(x) = 2, \quad \lim_{x \to \infty} f(x) = 2. \]
0210-3. Let \( f(s) = \sqrt[3]{s^4 + s} \).

Using the properties of limits, show that \( f \) is continuous at 5.

0210-4.

Let \( f(x) = \begin{cases} 
2x + 5, & \text{if } x < -1 \\
3, & \text{if } x = -1 \\
x^3 + 4, & \text{if } x > -1.
\end{cases} \)

a. Does \( \lim_{x \to -1} f(x) \) exist? If so, compute it.

b. Is \( f \) continuous at \(-1\)?
Let $g(x) = \begin{cases} 
\cos(2x), & \text{if } x < 0 \\
2, & \text{if } x = 0 \\
(x^3 - 1)^2, & \text{if } x > 0.
\end{cases}$

a. Does $\lim_{x \to 0} g(x)$ exist? If so, compute it.

b. Is $g$ continuous at 0?
Let \( g(x) = \begin{cases} 
\cos(2x), & \text{if } x < 0 \\
2, & \text{if } x = 0 \\
(x^3 - 1)^2, & \text{if } x > 0. 
\end{cases} \)

a. Does \( \lim_{x \to -1} g(x) \) exist? If so, compute it.

b. Is \( g \) continuous at \(-1\)?
0210-7. Let \( f(x) = x^{2/3} \).

a. Is \( f \) continuous at 0?
b. Is \( f \) continuous on \([0, \infty)\)?
c. Is \( f \) continuous?

0210-8. Let \( g(x) = x^{-2/3} \).

a. Is \( g \) continuous at 0?
b. Is \( g \) continuous on \((0, \infty)\)?
c. Is \( g \) continuous?

0210-9. Compute \( \lim_{x \to 64} \frac{x + \sqrt[3]{x}}{(x - 60)^2 + x - 46} \).
0210-10. Let \( f(x) = \begin{cases} 
  x^2 + 3, & \text{if } x < 2 \\
  2x + 2, & \text{if } 2 \leq x \leq 3 \\
  7[\cos(x - 3)], & \text{if } 3 < x. 
\end{cases} \)

a. At which numbers is the function \( f \) discontinuous?

b. For each of the numbers, given in Part a, where \( f \) is discontinuous, state whether or not \( f \) is continuous from the LEFT at that number.

c. For each of the numbers, given in Part a, where \( f \) is discontinuous, state whether or not \( f \) is continuous from the RIGHT at that number.
Let \( g(x) = \begin{cases} 
4e^x, & \text{if } x < 0 \\
8, & \text{if } x = 0 \\
7x + 4, & \text{if } 0 < x. 
\end{cases} \)

a. At which numbers is the function \( g \) discontinuous?

b. For each of the numbers, given in Part a, where \( g \) is discontinuous, state whether or not the discontinuity is removable.
0210-12. Find a number \( a \) s.t.

\[
f(w) = \begin{cases} 
  a e^w, & \text{if } w \leq 0 \\
  4aw^6 + 5a + 8, & \text{if } 0 < w
\end{cases}
\]

is continuous at \( w = 0 \).

0210-13. Let \( g(z) = \frac{2z^2 + 10z - 12}{z + 6} \).

Find a function \( q : \mathbb{R} \rightarrow \mathbb{R} \)

such that \( q \) is continuous at \(-6\)

and such that, \( \forall z \in \mathbb{R} \setminus \{-6\}, \quad q(z) = g(z) \).
0210-14.
Using the Intermediate Value Theorem, show that \(x^3 + x + 100001 = 0\) has a solution \(x = c\) that satisfies \(-101 < c < 101\).

0210-15.
Using the Intermediate Value Theorem, show that \(e^{3x} + \sin^2 x = x^2 - 1\) has a solution \(x = c\) that satisfies \(-\pi < c < \pi\).