CALCULUS
Maxima and minima
NEW
0450-1. a. Sketch the graph of a continuous function whose domain is \([2, \infty)\), and which has exactly two global minima, and exactly three local minima.

b. Sketch the graph of a continuous function whose domain is \((2, 4]\), and which has exactly two local minima, and exactly one global minimum and no global maxima.
Let \( f : [a, z] \to \mathbb{R} \) be the function whose graph is shown below.

i. For each number \( a, b, c, d, e, r, s, t, u, v, w, x, z \), state whether or not \( f \) has, at that number, a global maximum, a global minimum, a local maximum, or a local minimum.

ii. Which of \( a, b, c, d, e, r, s, t, u, v, w, x, z \) is a critical number for \( f \)?
0450-3. Let \( f : [-4, 4] \rightarrow \mathbb{R} \) be the function whose graph is displayed below.

a. At what numbers \( x \) does \( f(x) \) have a local minimum?

b. What are the corresponding local minimum values?

c. At what numbers \( x \) does \( f(x) \) have a local maximum?

d. What are the corresponding local maximum values?
0450-4. Let $f : [-4, 4] \to \mathbb{R}$ be the function whose graph is displayed below.

a. At what numbers $x$ does $f(x)$ have a global minimum?

b. What is the corresponding global minimum value?

c. At what numbers $x$ does $f(x)$ have a global maximum?

d. What is the corresponding global maximum value?
Sketch a graph of a continuous function whose domain is $(-3, 2)$ and which has no global minima, one local minimum, and two global maxima.

Sketch a graph of a function whose domain is $[1, 4]$, which is continuous on $[1, 4]$ and which has two global maxima, two global minima and one local minimum.
Define \( f : [-2, 2] \to \mathbb{R} \) by \( f(x) = -x^5 \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
Define $f : (-2, 2) \to \mathbb{R}$ by $f(x) = -x^5$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? 
   If so, at what number(s)?

c. Does $f$ have a global minimum? 
   If so, at what number(s)?

d. Does $f$ have a local maximum? 
   If so, at what number(s)?

e. Does $f$ have a local minimum? 
   If so, at what number(s)?

f. What are the critical numbers of $f$?
0450-9. Define $f : [-2, 2] \rightarrow \mathbb{R}$ by $f(x) = -3x$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
Define $f : (-2, 2) \rightarrow \mathbb{R}$ by $f(x) = -3x$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
Define \( f : (-2, 2) \rightarrow \mathbb{R} \) by \( f(x) = x^2 + x \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
Define \( f : [-2, 2] \rightarrow \mathbb{R} \) by
\[
f(x) = x^2 + x.
\]

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum?
   If so, at what number(s)?

c. Does \( f \) have a global minimum?
   If so, at what number(s)?

d. Does \( f \) have a local maximum?
   If so, at what number(s)?

e. Does \( f \) have a local minimum?
   If so, at what number(s)?

f. What are the critical numbers of \( f \)?
0450-13. Find the critical numbers of
\[ f(x) = x^3 + 3x^2 - 9x + 4. \]

0450-14. Find the critical numbers of
\[ f(x) = |x^2 - 2x - 8|. \]

0450-15. Find the critical numbers of
\[ f(x) = |x^2 + 6x + 9|. \]

0450-16. Find the critical numbers of
\[ f(x) = \cos(2x). \]

0450-17. Find the critical numbers of
\[ f(x) = |\cos(2x)|. \]
0450-18. Find the global maximum and minimum values of
\[ f(x) = x^3 - 15x^2 + 48x - 6 \]
on \( 0 \leq x \leq 5 \).

0450-19. Find the global maximum and minimum values of
\[ g(t) = \frac{3t}{t^2 + 9} \]
on \( -10 \leq t \leq 10 \).

0450-20. Find the global maximum and minimum values of
\[ H(s) = se^{-s^2/8} \]
on \( -1 \leq s \leq 8 \).