CALCULUS
Even more graphing problems
NEW
0500-1. Let $f : (0, 15) \setminus \{7\} \to \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   (ii) $f$ is decreasing;
   (iii) $f$ is concave up;
   and (iv) $f$ is concave down.

b. Find all points of inflection for $f$. 
0500-2. Let $f : (0, 15] \setminus \{7\} \to \mathbb{R}$ be continuous from the left at 15. The graph of $f'$ is shown below.

Find the maximal intervals on which

(i) $f$ is concave up;
and (ii) $f$ is concave down.
0500-3. Let $f : [0, 14) \to \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   and (ii) $f$ is decreasing.

b. Find all numbers at which
   (i) $f$ attains a local maximum;
   and (ii) $f$ attains a local minimum.
0500-4. Let $f$ be continuous on $[0, 14]$.

The graph of $f'$ is shown below.

a. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.

b. At what numbers does $f$ have
   (i) a local maximum?
   (ii) a local minimum?
0500-5. Let $f(x) = x^4 + 3x^3 + 3x^2 - 7$.

a. Find the maximal intervals on which (i) $f$ is increasing; and (ii) $f$ is decreasing.

b. Find all numbers at which (i) $f$ attains a local maximum; and (ii) $f$ attains a local minimum.

c. Find the maximal intervals on which (i) $f$ is concave up; and (ii) $f$ is concave down.
0500-6. Let \( f(x) = (9x^2 - 6x + 2)e^{-3x} \).

**a.** Find the maximal intervals on which
(i) \( f \) is increasing;
and (ii) \( f \) is decreasing.

**b.** Find all numbers at which
(i) \( f \) attains a local maximum;
and (ii) \( f \) attains a local minimum.

**c.** Find the maximal intervals on which
(i) \( f \) is concave up;
and (ii) \( f \) is concave down.

**d.** Find all points of inflection for \( f \).
0500-7. Let \( f(x) = xe^{8x^3/3} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), use the Second Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
Let \( f(x) = x^4 e^{-x^2/2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), what does the Second Derivative Test tell you about that critical number?

c. For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
0500-9. Sketch the graph of a function

\[ H : [0, 8] \rightarrow \mathbb{R} \]

with the following properties:

- (●) \( H \) is continuous on \([0, 8]\);
- (●) \( H'' \) is continuous on \((0, 8)\);
- (●) \( H(0) = H(4) = 0; \ H(8) = 2; \)
- (●) \( H'(2) = H'(6) = 0; \)
- (●) \( H'' > 0 \) on \((0, 4)\);

and (●) \( H'' > 0 \) on \((5, 8)\).

0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \)

s.t. \( g \) attains a local min value of 20 at \(-1\)

and a local max value of \(-88\) at \(1\).
0500-11. Let \( f(x) = \tan^3(x/2) \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).
c. Find all max intervals of incr/decr for \( f \).
d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = \ln(5 - x^2) \).

a. **Describe** the symmetries, if any, of \( f \).

b. **Find all max** intervals of pos/neg for \( f \).  
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. **Find all max** intervals of incr/decr for \( f \).

d. **Find all max** intervals of cc up/cc dn for \( f \).

e. **Sketch** the graph of \( f \).
Let \( f(x) = \frac{2x}{\sqrt{5 - x^2}} \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).
d. Find all max intervals of cc up/cc dn for \( f \).
e. Sketch the graph of \( f \).
0500-14. Let \( f(x) = x^4 + 5x^2 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-15. Let \( f(x) = \frac{1}{x^3 - x} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = \sqrt{x^2 + 4x + 4} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-17. Let \( f(x) = 2x + \pi + \cos x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-18. Let \( f(x) = x^2 e^{-x^2/8} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-19. Let \[ f(x) = \frac{x^2 - 4x + 3}{x - 1}. \]

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).