CALCULUS
Definite integration and Riemann sum problems
NEW
0590-1. Let $f(x) = 2 + 2x^2$.

a. Compute $L_4 S^2_{-2} f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $L_4 S^2_{-2} f$.

b. Compute $M_4 S^2_{-2} f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $M_4 S^2_{-2} f$.

c. Compute $R_4 S^2_{-2} f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $R_4 S^2_{-2} f$. 
0590-2. Let \( f(x) = e^x + 6 \).

a. Compute \( L_2 S_0^8 f \) to three decimal places.
b. Compute \( M_2 S_0^8 f \) to three decimal places.
c. Compute \( R_2 S_0^8 f \) to three decimal places.

0590-3. Let \( f(x) = \sin^2 x \).

a. Compute \( L_3 S_0^{2\pi} f \) to three decimal places.
b. Compute \( M_3 S_0^{2\pi} f \) to three decimal places.
c. Compute \( R_3 S_0^{2\pi} f \) to three decimal places.
A car’s acceleration is positive from time 0 to time 24 seconds, and its velocity at various times is given in the table below.

<table>
<thead>
<tr>
<th>time (secs)</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>40</td>
<td>56</td>
<td>68</td>
<td>77</td>
<td>81</td>
<td>83</td>
</tr>
</tbody>
</table>

Find upper and lower estimates for the distance traveled by the car over these 24 seconds.
The gph of a function $f$ appears below.

Estimate $\int_0^{10} f(x) \, dx$ by computing

(a) $L_5 S_0^{10} f$,  
(b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.  

5
0590-6. Express the area under $y = e^{-x^2/5}$ from $x = -2$ to $x = 0$ as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-7. Express the area under $y = \sqrt{x^3 + x + 5}$ from $x = 1$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don’t evaluate the limit.)

0590-8. Express the area under $y = \cos(x^4 - x)$ from $x = 0$ to $x = 5$ as a limit of right endpt Riemann sums. (Don’t evaluate the limit.)
0590-9. Express \[ \int_{2}^{4} \frac{e^{-x^2}}{\sqrt{\pi}} \, dx \] as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-10. Let \( f(x) = 2x^3 \).

a. Write \( R_nS_0^2 f \) as a rational expression in \( n \) (i.e., as one polynomial in \( n \) divided by another).

b. Compute \( \lim_{n \to \infty} R_nS_0^2 f \).
The limit
\[
\lim_{{n \to \infty}} \left[ \frac{3}{n} \sum_{{j=1}}^{n} \left( \cos^2 \left( -4 + j \left( \frac{3}{n} \right) \right) \right) \right]
\]
represents the area under \( y = f(x) \) from \( x = a \) to \( x = b \), for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.
The limit
\[
\lim_{n \to \infty} \left[ \frac{5}{n} \sum_{j=0}^{n-1} \left( \cos \left( \frac{1}{3 + j(5/n)} \right) \right) \right]
\]
represents the area under \( y = f(x) \) from \( x = a \) to \( x = b \), for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.
0590-13. Let \( f(x) = 2 - \sqrt{4 - x^2} \).

a. Sketch the graph of \( y = f(x) \).

b. Compute \( \int_{-2}^{2} f(x) \, dx \), by interpreting this integral as an area.