CALCULUS
The limit game and
the exact definition of a limit
NEW
0150-1. For the function $g$ graphed below, what is the largest number $\delta$ such that
\[ 0 < |s - 4| < \delta \quad \Rightarrow \quad |(g(s)) - 5| < 0.6 \] ?

**ANSWER:**

\[ 4 - 3.3 = 0.7 \]
and \[ 6.5 - 4 = 2.5 \]

Since $\delta$ must be smaller than both, the largest possibility is \[ \delta = 0.7. \]
Let \( f(x) = -2x + 8 \).

Show a graph of \( y = f(x) \) that includes the points \((2, 4), (3, 2)\) and \((4, 0)\).

Find the largest number \( \delta \) such that
\[
|x - 3| < \delta \quad \Rightarrow \quad |(f(x)) - 2| < 0.6.
\]

ANSWER:

[Graph showing the function \( y = f(x) \) with points \((2, 4), (3, 2), (4, 0)\) marked.]

The largest possibility: \( \delta = 0.3 \)
0150-3. Let \( g(x) = [-2x + 8] \left[ \frac{x - 3}{x - 3} \right] \).

Show a graph of \( y = g(x) \) that includes the points \((2, 4)\) and \((4, 0)\).

Find the largest number \( \delta \) such that
\[
0 < |x - 3| < \delta \quad \Rightarrow \quad |(g(x)) - 2| < 0.6.
\]

**ANSWER:**

\[
\text{run} = \frac{\text{rise}}{\text{slope}} = \frac{\pm 0.6}{-2} = \mp 0.3
\]

The largest possibility: \( \delta = 0.3 \)
In shop class, you are asked to build a square sheet of metal of area 169 square inches. The area can be slightly off, but must be between 164 and 174 square inches. Say you have access to a machine that will punch out a perfect square, and the side length (in inches) is controlled by a dial. **How close** to 13 must you set the dial to get the area to be in the specified range? **Give** your answer to five decimal places.

**ANSWER:**

\[
164 < s^2 < 174
\]

\[
12.80625 \leq \sqrt{164} < s < \sqrt{174} \leq 13.19091
\]

\[
13.19091 - 13 = 0.19091
\]

\[
13 - 12.80625 = 0.19375
\]

Must be within 0.19091.
0150-5. Prove that \( \lim_{x \to 6} -4x = -24 \).

Your writeup should read:

Given \( \varepsilon > 0 \).
Let \( \delta = \cdots \).
Assume \( 0 < |x - 6| < \delta \).
Then \( |(-4x) - (-24)| < 4\delta \).
Then \( |(-4x) - (-24)| < \varepsilon \).

All you need do is fill in the ellipsis (\( \cdots \)) with a carefully chosen expression of \( \varepsilon \).

**Hint:** The last sentence in the writeup clearly follows from the penultimate sentence if \( 4\delta = \varepsilon \).

**Answer:** \( \delta = \varepsilon / 4 \)