CALCULUS
Properties of the definite integral
NEW
Which of the following is true?

a. \( \int_{\pi}^{2\pi} \sin^4 x \leq 0 \)

b. \( \int_{\pi}^{2\pi} \sin^4 x \geq 0 \)

Explain your choice, invoking the properties of integrals.

**ANSWER:** \( \forall u \in \mathbb{R}, \) we have \( u^4 \geq 0, \)
so \( \forall x \in \mathbb{R}, \) we have \( \sin^4 x \geq 0. \)

Then \( \int_{\pi}^{2\pi} \sin^4 x \, dx \geq 0, \) i.e., b is true.
Which of the following is true?

a. \( \int_{-2}^{-3} e^{-x} \, dx \leq \int_{-2}^{-3} e^{-2x} \, dx \)

b. \( \int_{-2}^{-3} e^{-x} \, dx \geq \int_{-2}^{-3} e^{-2x} \, dx \)

Explain your choice, invoking the properties of integrals.

ANS: On \(-3 \leq x \leq -2\), \(x \geq 2x\),

so \(-x \leq -2x\), so \(e^{-x} \leq e^{-2x}\).

Then \(\int_{-3}^{-2} e^{-x} \, dx \leq \int_{-3}^{-2} e^{-2x} \, dx\),

so \(-\int_{-3}^{-2} e^{-x} \, dx \geq -\int_{-3}^{-2} e^{-2x} \, dx\),

i.e., \(\int_{-2}^{-3} e^{-x} \, dx \geq \int_{-2}^{-3} e^{-2x} \, dx\), i.e., b is true.