CALCULUS
Volume by cylindrical shells:
Problems
NEW
Using the shell method, find the volume in a ball of radius 13, following the diagram shown below.
Using the shell method, find the volume in a ball of radius 13, following the diagram shown below.

\[ y = \sqrt{13^2 - x^2} \]

\[ \int_{0}^{13} \left[ 2\pi y \right] \left[ 2\sqrt{13^2 - y^2} \right] \, dy \]
Using the shell method, find the volume in a ball of radius 13, following the diagram . . .

**ANSWER:** \( z = 13^2 - y^2, \quad dz = -2y \, dy \)

\[
\int_0^{13} [2\pi y] \left[ 2\sqrt{13^2 - y^2} \right] \, dy = -2\pi \int_0^{13^2} \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{13^2} \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{13^2} z^{1/2} \, dz
\]

\[
= 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \to 0}^{13^2}
\]
0750-1. Using the shell method, find the volume in a ball of radius 13, following the diagram . . .

**ANSWER:** \( z = 13^2 - y^2, \quad dz = -2y \, dy \)

\[
\int_0^{13} [2\pi y] \left[ 2\sqrt{13^2 - y^2} \right] \, dy = 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \to 0}^{z: \to 13^2}
\]

\[
= 2\pi \left[ \frac{(13^2)^{3/2}}{3/2} - 0 \right]
\]

\[
= \frac{4}{3} \pi \left[ 13^3 \right]
\]
We create a napkin holder by drilling a cylindrical hole of radius 5 through the middle of a ball of radius 13, as shown below. Using the shell method, find its volume.
0750-2. We create a napkin holder by drilling a cylindrical hole of radius 5 through the middle of a ball of radius 13, as shown below. Using the shell method, find its volume.

**ANSWER:** \[ \int_{5}^{13} \left[ 2\pi y \right] \left[ 2\sqrt{13^2 - y^2} \right] \, dy \]
0750-2. We create a napkin holder . . .

Using the shell method, find its volume.

**ANSWER:** \( z = 13^2 - y^2 \), \( dz = -2y \, dy \)

\[
\int_5^{13} 2\pi y \left(2\sqrt{13^2 - y^2}\right) \, dy = -2\pi \int_{13^2-5^2}^0 \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{13^2-5^2} \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{12^2} z^{1/2} \, dz
\]

\[
= 2\pi \left. \left[ \frac{z^{3/2}}{3/2} \right] \right|_{z:0}^{z:12^2} = 9
\]
NEW

0750-2. We create a napkin holder ... Using the shell method, find its volume.

**ANSWER:** \[ z = 37^2 - y^2, \quad dz = -2y \, dy \]

\[
\int_{12}^{37} [2\pi y] \left[ 2\sqrt{37^2 - y^2} \right] \, dy = 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \rightarrow 0}^{z: \rightarrow 12^2} \\
= 2\pi \left[ (12^2)^{3/2} \right] \\
= \frac{4}{3} \pi \left[ 12^3 \right]
\]
Let \( R \) be the region bounded by
\[
y = \frac{1}{5}(x - 2)^2(x - 4)^2 + \frac{6}{5} \quad \text{and} \quad y = 3.
\]

a. Sketch \( R \).

b. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the \( x \)-axis. **Do not evaluate** the integral.

c. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the \( y \)-axis. **Do not evaluate** the integral.

d. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the line \( x = \frac{1}{2} \). **Do not evaluate** the integral.
Let $R$ be the region bounded by $y = \frac{1}{5}(x - 2)^2(x - 4)^2 + \frac{6}{5}$ and $y = 3$.

**ANSWER:**

a. Sketch $R$. 

![Graph showing the region $R$ bounded by the given functions and the line $y = 3$.](image_url)
Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the $x$-axis. Do not evaluate the integral.

$$b. \int_{1}^{5} \pi (3)^2 - \pi \left( \frac{1}{5}(x - 2)^2(x - 4)^2 + \frac{6}{5} \right)^2 \, dx$$
Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the $y$-axis. Do not evaluate the integral.

\[ \int_{1}^{5} [2\pi x] \left[ 3 - \left( \frac{1}{5}(x - 2)^2(x - 4)^2 + \frac{6}{5} \right) \right] \, dx \]
Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the line $x = \frac{1}{2}$. Do not evaluate the integral.

$$\int_1^5 \left[ 2\pi \left( x - \frac{1}{2} \right) \right] \left[ 3 - \left( \frac{1}{5} (x - 2)^2 (x - 4)^2 + \frac{6}{5} \right) \right] \, dx$$
Let $R$ be the region bounded by $x = 1 + e^{-y^2}$, $x = 1$, $y = 0$ and $y = 2$.

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**ANSWER:**

a. [Diagram of the region $R$]
Let $R$ be the region bounded by

$$x = 1 + e^{-y^2}, \ x = 1, \ y = 0 \text{ and } y = 2.$$ 

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**ANSWER:**

b. \[ \int_{0}^{2} [2\pi y] \left[ 1 + e^{-y^2} - 1 \right] \, dy \]

\[ = 2\pi \left[ \int_{0}^{2} ye^{-y^2} \, dy \right] \]

\[ = 2\pi \left[ \int_{0}^{-4} e^u \frac{du}{-2} \right] = -\pi \left[ \int_{0}^{-4} e^u \, du \right] \]

\[ = -\pi [e^u]_{u: \to -4}^{u: \to 0} = -\pi [e^{-4} - 1] \]

\[ = \pi \left[ 1 - e^{-4} \right] \]
Let $R$ be the region bounded by $x = y^2 + y$, $x = 0$ and $y = 1$.

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$. 
Let $R$ be the region bounded by 

$$x = y^2 + y, \ x = 0 \text{ and } y = 1.$$ 

a. Sketch $R$. 

ANS: a.
b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

**ANSWER:** b. $\int_0^1 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy$

$$= \pi \int_0^1 \left[ 1 + y^4 + y^2 + 2y^2 + 2y + 2y^3 \right] - 1 \, dy$$
b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

**ANSWER:** b. $\int_0^1 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy$

$$= \pi \int_0^1 \left[ 1 + y^4 + y^2 + 2y^2 + 2y + 2y^3 \right] - 1 \, dy$$

$$= \pi \int_0^1 y^4 + 2y^3 + 3y^2 + 2y \, dy$$

$$= \pi \left[ \frac{y^5}{5} + \frac{y^4}{2} + y^3 + y^2 \right]_{y: \to 1}^{y: \to 0}$$
Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

**ANSWER:** 

$$
\int_0^1 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy
$$

\[
= \pi \left[ \frac{y^5}{5} + \frac{y^4}{2} + y^3 + y^2 \right]_{y: \to 1} - \pi \left[ 1^2 \right]_{y: \to 0}
\]

\[
= \pi \left[ \frac{1^5 - 0^5}{5} + \frac{1^4 - 0^4}{2} + 1^3 - 0^3 + 1^2 - 0^2 \right]
\]

\[
= \pi \left[ \frac{1}{5} + \frac{1}{2} + 1 + 1 \right] = \frac{27\pi}{10}
\]
Let $R$ be the region bounded by $x = \sin y$, $x = 0$, $y = \pi/8$, and $y = 3\pi/4$. Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating $R$ about the line $y = 3\pi/2$.

**ANS:**

$$y = 3\pi/2$$

$$x = \sin y$$

$$y = 3\pi/4$$

$$y = \pi/8$$

$$\text{VOLUME} = \int_{\pi/8}^{3\pi/4} 2\pi \left( \frac{3\pi}{2} - y \right) \sin y \, dy$$
NEW 0750-7. Describe the solid of revolution whose volume is given by

\[ 2\pi \int_2^5 x \left[ (e^{4x}) - (\sin(\pi x)) \right] \, dx. \]

Do not evaluate this integral.

**ANSWER:**

This is the solid of revolution obtained by revolving, about the y-axis, the region bounded by

\[ y = e^{4x}, \ y = \sin(\pi x), \ x = 2 \text{ and } x = 5. \]

**NOTE:** This is the most natural answer, given the problem, but other answers are correct.
0750-8. Describe the solid of revolution whose volume is given by

\[ 2\pi \int_{2}^{5} [x + 6] \left[ (e^{4x}) - (\sin(\pi x)) \right] \, dx. \]

Do not evaluate this integral.

**ANSWER:**

This is the solid of revolution obtained by revolving, about the line \( x = -6 \), the region bounded by

\[ y = e^{4x}, \quad y = \sin(\pi x), \quad x = 2 \text{ and } x = 5. \]

**NOTE:** This is the most natural answer, given the problem, but other answers are correct.