CALCULUS
Average rates of change
NEW
Water is being drained from a tub; the amount in the tub is constantly monitored, and is tabulated against time as follows:

<table>
<thead>
<tr>
<th>hrs</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>gallons</td>
<td>101</td>
<td>80</td>
<td>35</td>
<td>26</td>
</tr>
</tbody>
</table>

Let $W$ be the amount in the tank at time $t$. Let $B = (9, 35)$, a point on the graph of $W$.

a. Find the slope of the secant lines between $B$ and the other points on the graph of $W$ appearing in the table above.

b. Estimate the slope of the tangent line to the graph of $W$ at the point $B$, by averaging the following two numbers: the slope of the secant line between $B$ and $(6, 80)$ and the slope of the secant line between $B$ and $(12, 26)$. 

Let $A$ be the point $(1, 1)$ on the graph of $y = 3 - 2x^4$. Let $B$ be a variable point $(x, 3 - 2x^4)$ on the same graph.

a. **Compute** the slope of the secant line between $A$ and $B$, when $x$ is equal to:

(i) 2  (ii) 1.1  (iii) 1.01
(iv) 0  (v) 0.9  (vi) 0.99
(vii) $1 + h$, with $h \neq 0$

b. **Guess** the slope of the tangent line to $y = 3 - 2x^4$ at $A$.

c. Using b, **write** an equation of the tangent line to $y = 3 - 2x^4$ at $A$. 
A tennis player, in a fit of rage over a lost point, throws his racquet into the air. Assume that its distance, in feet, above the ground, \( t \) seconds later, is \( 5 + 60t - 16t^2 \).

a. Find its average velocity over the time period starting at time 4, and continuing for the following number of seconds:

(i) 1  
(ii) 0.5  
(iii) 0.01  
(iv) 0.001  
(v) 0.0001  
(vi) 0.00005  
(vii) \( \Delta t \), with \( \Delta t \neq 0 \)

b. Guess its instantaneous velocity 4 seconds after it’s thrown.