CALCULUS
Maxima and minima
NEW
a. Sketch the graph of a continuous function whose domain is \([1, \infty)\), and which has exactly two global maxima, and exactly one local minimum.

b. Sketch the graph of a continuous function whose domain is \([2, 4)\), and which has exactly two global minima, and no global maxima.
0450-2. Let \( f : [a, z] \to \mathbb{R} \) be the function whose graph is shown below.

i. For each number \( a, b, c, d, e, r, s, t, u, v, w, x, z \), state whether or not \( f \) has, at that number, 
   - a global maximum
   - a global minimum
   - a local maximum
   - a local minimum.

ii. Which of \( a, b, c, d, e, r, s, t, u, v, w, x, z \) is a critical number for \( f \)?
0450-3. Let \( f : [-4, 4] \rightarrow \mathbb{R} \) be the function whose graph is displayed below.

a. At what numbers \( x \) does \( f(x) \) have a local minimum?

b. What are the corresponding local minimum values?

c. At what numbers \( x \) does \( f(x) \) have a local maximum?

d. What are the corresponding local maximum values?
0450-4. Let $f : [-4, 4] \rightarrow \mathbb{R}$ be the function whose graph is displayed below.

a. At what numbers $x$ does $f(x)$ have a global minimum?

b. What is the corresponding global minimum value?

c. At what numbers $x$ does $f(x)$ have a global maximum?

d. What is the corresponding global maximum value?
0450-5. Sketch a graph of a function whose domain is $[-3, 2]$ and which has no global minima, one local minimum, and two global maxima.

Note: By the Extreme Value Theorem, it cannot be continuous on $[-3, 2]$.

0450-6. Sketch a graph of a function whose domain is $[1, 4]$, which is continuous on $[1, 4]$ and which has three global maxima, one local maximum and two global minima.
0450-7. Define \( f : [-2, 2] \rightarrow \mathbb{R} \) by \( f(x) = -x^6 \).

a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
0450-8. Define $f : (-2, 2) \to \mathbb{R}$ by $f(x) = -x^6$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
Define $f : [-2, 2] \rightarrow \mathbb{R}$ by $f(x) = -3x^5$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
Define $f : (-2, 2) \to \mathbb{R}$ by $f(x) = -3x^5$.

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
Define \( f : (-2, 2) \to \mathbb{R} \) by
\[
f(x) = x^3 - x.
\]
a. Sketch the graph of \( f \).

b. Does \( f \) have a global maximum? If so, at what number(s)?

c. Does \( f \) have a global minimum? If so, at what number(s)?

d. Does \( f \) have a local maximum? If so, at what number(s)?

e. Does \( f \) have a local minimum? If so, at what number(s)?

f. What are the critical numbers of \( f \)?
Define $f : [-2, 2] \to \mathbb{R}$ by
$$f(x) = x^3 - x.$$

a. Sketch the graph of $f$.

b. Does $f$ have a global maximum? If so, at what number(s)?

c. Does $f$ have a global minimum? If so, at what number(s)?

d. Does $f$ have a local maximum? If so, at what number(s)?

e. Does $f$ have a local minimum? If so, at what number(s)?

f. What are the critical numbers of $f$?
0450-13. Find the critical numbers of
\[ f(x) = x^3 + 4x^2 - 3x + \sqrt{7}. \]

0450-14. Find the critical numbers of
\[ f(x) = |x^2 - 4x + 3|. \]

0450-15. Find the critical numbers of
\[ f(x) = |4x^2 - 4x + 1|. \]

0450-16. Find the critical numbers of
\[ f(x) = \sin(4x). \]

0450-17. Find the critical numbers of
\[ f(x) = |\sin(4x)|. \]
0450-18. Find the global maximum and minimum values of
\[ f(x) = 3x^3 - x + \pi \]
on \(0 \leq x \leq 1\).

0450-19. Find the global maximum and minimum values of
\[ g(t) = \frac{2t}{t^2 + 4} \]
on \(-3 \leq t \leq 0\).

0450-20. Find the global maximum and minimum values of
\[ H(s) = se^{-s^2/2} \]
on \(0 \leq s \leq 3\).