CALCULUS
Volume by slices and
the disk and washer methods:
Problems
NEW
0720-1. Let $R$ be the region bounded by $y = x + 2$ and $x = 4$ in $2 \leq y \leq 3$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

0720-2. Let $R$ be the region bounded by $y = x^2$ and $y = 2x$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the line $x = -2$. 

$^2$
0720-3. Let $R$ be the region bounded by $y = \ln x$, $x = 9$ and $y = 1$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

0720-4. Let $R$ be the region bounded by $y = \cos x$ and $y = 0$ in $0 \leq x \leq \frac{\pi}{3}$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

Hint: $\cos^2 x = \frac{1 + \cos(2x)}{2}$
0720-5. Let $R$ be the region bounded by
\[(x - 1)^2 + (y - 4)^2 = 9.\]

a. Sketch $R$.
b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that $2 \int_{-3}^{3} \sqrt{9 - u^2} \, du$ is known; it is the area enclosed in a circle of radius 3.
0720-6. Let \( R \) be the region bounded by 
\[ y = x^5 \text{ and } x = y^6. \]

a. Sketch \( R \).
b. Find the volume of the solid obtained by rotating \( R \) about the line \( y = -1/2 \).

c. Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).

0720-7. Let \( R \) be the region bounded by 
\[ y = x^4 \text{ and } x = y^6. \]

a. Sketch \( R \).
b. Find the volume of the solid obtained by rotating \( R \) about the line \( y = -1/2 \).

c. Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).
0720-8. Let $R$ be the region bounded by $x = 2 + e^y$, $x = \sin y$ in $0 \leq y \leq \pi/2$. Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating $R$ about the line $x = 6$.

0720-9. Describe the solid of revolution whose volume is given by

$$
\pi \int_{1}^{3/2} (9e^{2y} - 2) \, dy.
$$

Do not evaluate this integral.

0720-10. Describe the solid of revolution whose volume is given by

$$
\pi \int_{0}^{\pi/2} (4 + \cos y)^2 - 16 \, dy.
$$

Do not evaluate this integral.
A solid $S$ sits above a horizontal plane $P$. $\forall x \geq 0$, let $P_x$ be the horizontal plane that is $x$ units above $P$. Suppose that $S$ lies between $P_1$ and $P_2$. Suppose, also, that $\forall x \in [1, 2]$, the intersection of $S$ and $P_x$ is the region inside a rectangle whose base has length $5x$ and whose altitude has length $e^{3x^2}$.

Compute the volume of $S$. 
0720-12. Using the disk method, find the volume in a ball of radius 26, following the diagram shown below.
We create a napkin holder by drilling a cylindrical hole of radius 10 through the middle of a ball of radius 26, as shown below. Using the washer method, find its volume.