CALCULUS
Derivatives of trigonometric functions
NEW
0360-1. Differentiate \( f(x) = ex^9 + 4 \tan x \).

**ANSWER:** \( f'(x) = 9ex^8 + 4 \sec^2 x \)

0360-2. Differentiate \( u(t) = -(\sin(6))t^8 + 6e^t + e^2 - \csc t \).

**ANSWER:**

\[
\begin{align*}
u'(t) &= -8(\sin(6))t^7 + 6e^t + 0 + (\csc t)(\cot t) \\
&= -8(\sin(6))t^7 + 6e^t + (\csc t)(\cot t)
\end{align*}
\]

0360-3. Differentiate \( p(t) = t^3 \sec t \).

**ANSWER:**

\[
p'(t) = (3t^2)(\sec t) + (t^3)(\sec t)(\tan t)
\]
**0360-4.** Differentiate \( Q(s) = \frac{-\pi e^5 - \sec s}{(\csc s)(\cot s)} \).

**ANSWER:** \( Q'(s) = \frac{[(\csc s)(\cot s)][-(\sec s)(\tan s)] - [\pi e^5 - \sec s][-(\csc s)(\cot^2 s) - (\csc^3 s)]}{[(\csc s)(\cot s)]^2} \)

**0360-5.** Differentiate \( F(x) = \frac{x^2 e^x - \cos x}{e^x \tan x} \).

**ANSWER:** \( F'(x) = \frac{[e^x \tan x][2xe^x + x^2 e^x + \sin x] - [x^2 e^x - \cos x][(e^x \tan x) - (e^x \sec^2 x)]}{[e^x \tan x]^2} \)
0360-6. Find an equation of the tangent line to the graph of \( y = \frac{6e^{-\pi/2}e^x - \cos x}{2e^{-\pi/2}e^x \csc x} \)
at the point \((\pi/2, 3)\).

**ANSWER:**

\[
\frac{d}{dx} \left( \frac{6e^{-\pi/2}e^x - \cos x}{2e^{-\pi/2}e^x \csc x} \right) = \left[ \frac{2e^{-\pi/2}e^x \csc x}{2e^{-\pi/2}e^x \csc x} \right]^2 \\
\left[ \frac{d}{dx} \left( \frac{4e^{-\pi}e^x - \tan x}{2e^{-\pi}e^x \cos x} \right) \right]_{x \to \pi/2} = \\
\left[ \frac{2e^{-\pi/2}e^{\pi/2} \csc(\pi/2)}{6e^{-\pi/2}e^{\pi/2} + \sin(\pi/2)} - \frac{6e^{-\pi/2}e^{\pi/2} - \cos(\pi/2)}{2e^{-\pi/2} \left[ e^{\pi/2}(\csc(\pi/2)) - e^{\pi/2}(\csc(\pi/2))(\cot(\pi/2)) \right]} \right] \\
\left[ 2e^{-\pi/2}e^{\pi/2} \csc(\pi/2) \right]^2
Find an equation of the tangent line to the graph of \( y = \frac{6e^{-\pi/2}e^x - \cos x}{2e^{-\pi/2}e^x \csc x} \) at the point \( (\pi/2, 3) \).

\[
\text{ANSWER: } \left[ \frac{d}{dx} \left( \frac{4e^{-\pi/2}e^x - \tan x}{2e^{-\pi/2}e^x \cos x} \right) \right]_{x \to \pi/2} = \\
\left[ \frac{2 \csc(\pi/2)[6 + \sin(\pi/2)] - [6 - \cos(\pi/2)][2 \left( \csc(\pi/2) - (\csc(\pi/2))(\cot(\pi/2)) \right)]}{2 \csc(\pi/2)^2} \right] = \\
\frac{[2(1)][6 - 1] - [6 - 0][2[1 - (1)(0)]]}{[2(1)]^2}
\]
0360-6. Find an equation of the tangent line to the graph of 
\[ y = \frac{6e^{-\pi/2}e^x - \cos x}{2e^{-\pi/2}e^x \csc x} \]
at the point \((\pi/2, 3)\).

**ANSWER:**
\[
\left[ \frac{d}{dx} \left( \frac{4e^{-\pi}e^x - \tan x}{2e^{-\pi}e^x \cos x} \right) \right]_{x \to \pi} = \\
= \frac{[2(1)] [6 - 1] - [6 - 0] [2 [1 - (1)(0)]]}{[2(1)]^2} \\
= \frac{[2] [5] - [6] [2]}{4} = -\frac{2}{4} = -\frac{1}{2}
\]
equation: \[ y = 3 - \left( \frac{1}{2} \right) (x - (\pi/2)) \]
A laser pointer, resting on the ground, is casting red light on a blue wall that is 18 ft away, as in the diagram. It is being turned upward, and its angle with the ground is denoted $\alpha$ (radians). Let $y$ denote the distance from the point of light on the wall straight down to the ground.

a. Find a formula for $y$ in terms of $\alpha$.

b. At the moment when $\alpha = 2\pi/9$, 
   i. compute $y$ and 
   ii. compute how fast $y$ is changing with respect to $\alpha$. 
0360-7. a. Find a formula for $y$ in terms of $\alpha$.
b. At the moment when $\alpha = 2\pi / 9$,
   i. compute $y$ and
   ii. compute how fast $y$ is changing with respect to $\alpha$.

**ANSWER:**

a. $\tan \alpha = y/18$, so $y = 18 \tan \alpha$.

b. i. $[y]_{\alpha \rightarrow 2\pi / 9} = 18 \left[ \tan \left( \frac{2\pi}{9} \right) \right]$
   $\approx 18[0.83910] \approx 15.1$ ft

   ii. $dy/d\alpha = 18 \sec^2 \alpha$

   $[dy/d\alpha]_{\alpha \rightarrow 2\pi / 9} = 18 \left[ \sec^2 \left( \frac{2\pi}{9} \right) \right]$
   $\approx 18 (1.30541)^2 \approx 30.7$ ft/radian