CALCULUS
Even more graphing problems
NEW
0500-1. Let \( f : (0, 15) \setminus \{7\} \to \mathbb{R} \) be as shown. 

a. Find the maximal intervals on which 
   
   (i) \( f \) is increasing;  
   
   (ii) \( f \) is decreasing;  
   
   (iii) \( f \) is concave up;  

   and (iv) \( f \) is concave down. 

b. Find all points of inflection for \( f \).
0500-1. Let \( f : (0, 15) \backslash \{7\} \rightarrow \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   
   (i) \( f \) is increasing;
   (ii) \( f \) is decreasing;

ANSWER:

a. (i) incr on \((0, 3]\), on \([5, 7)\) and on \([9, 13]\)

a. (ii) decr on \([3, 5]\), on \((7, 9]\) and on \([13, 15]\)
0500-1. Let \( f : (0, 15) \setminus \{7\} \rightarrow \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (iii) \( f \) is concave up;
   and (iv) \( f \) is concave down.

b. Find all points of inflection for \( f \).

ANS:  
   a. (iii) cc up on \([4, 7)\) and on \((7, 11]\)
   a. (iv) cc dn on \((0, 4]\) and on \([11, 15)\)
   b. points of inflection: \((4, 1)\) and \((11, 1)\)
0500-2. Let $f : [0, 15) \setminus \{7\} \to \mathbb{R}$ be continuous from the right at 0. The graph of $f'$ is shown below. Find the maximal intervals on which 

(i) $f$ is concave up; 
and (ii) $f$ is concave down.

**ANSWER:**
(i) $f$ cc up on $[0, 3]$, on $[5, 7)$ and on $[9, 13]$ 
(ii) $f$ cc dn on $[3, 5]$, on $(7, 9]$ and on $[13, 15]$
Let $f : (0, 14] \rightarrow \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   and (ii) $f$ is decreasing.

b. Find all numbers at which
   (i) $f$ attains a local maximum;
   and (ii) $f$ attains a local minimum.
0500-3. Let \( f : (0, 14] \to \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

**ANSWER:** a.

(i) \( f \) is incr on \((1, 5]\) and on \([12, 14]\).
(ii) \( f \) is decr on \([5, 12]\).
Let $f : (0, 14] \to \mathbb{R}$ be as shown.

b. Find all numbers at which
   (i) $f$ attains a local maximum;
   and (ii) $f$ attains a local minimum.

**ANSWER:** b.
   (i) $f$ attains a local maximum at 5.
   (ii) $f$ attains a local minimum at 13.
0500-4. Let \( f \) be continuous on \((0, 14]\).
The graph of \( f' \) is shown below.

a. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.

b. At what numbers does \( f \) have
   (i) a local maximum?
   (ii) a local minimum?
0500-4. Let $f$ be continuous on $(0,14]$.
The graph of $f'$ is shown below.

a. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.

**ANSWER:** a.
(i) $f$ is cc up on $(0,6]$.
(ii) $f$ is cc down on $[6,14]$. 
0500-4. Let $f$ be continuous on $(0, 14]$. The graph of $f'$ is shown below.

b. At what numbers does $f$ have
   (i) a local maximum? (ii) a local minimum?

**ANSWER:** b.

(i) $f$ has a local maximum at 9.
(ii) $f$ has a local minimum at 2.
0500-5. Let \( f(x) = 2x^4 + 8x^3 + 9x^2 + 9 \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.
0500-5. Let \( f(x) = 2x^4 + 8x^3 + 9x^2 + 9 \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

\[ f'(x) = 8x^3 + 24x^2 + 18x \]
\[ = 2x(2x + 3)^2 \]

a.(i) \( f \) is increasing on \([0, \infty)\).
   (ii) \( f \) is decreasing on \((-\infty, 0]\).

b.(i) \( f \) does not attain a local maximum.
   (ii) \( f \) attains a local minimum at 0.
Let \( f(x) = 2x^4 + 8x^3 + 9x^2 + 9 \).

**c. Find** the maximal intervals on which 
(i) \( f \) is concave up; and 
(ii) \( f \) is concave down.

**ANSWER:**
\[
f'(x) = 8x^3 + 24x^2 + 18x
\]
\[
f''(x) = 24x^2 + 48x + 18
= 6(2x + 3)(2x + 1)
\]

roots of \( f'' \): \(-\frac{3}{2}\) and \(-\frac{1}{2}\)

**c.**
(i) \( f \) is concave up on \((-\infty, -\frac{3}{2}]\) and on \([-\frac{1}{2}, \infty)\). 
(ii) \( f \) is concave down on \([-\frac{3}{2}, -\frac{1}{2}]\).
Let \( f(x) = (x^2 + 2x + 3)e^x \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.

d. Find all points of inflection for \( f \).
Let \( f(x) = (x^2 + 2x + 3)e^x \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

**Answer:**

\[
f'(x) = (2x + 2)e^x + (x^2 + 2x + 3)e^x
= (x^2 + 4x + 5)e^x
\]

\[4^2 - 4 \cdot 1 \cdot 5 < 0, \text{ so } \forall x \in \mathbb{R}, x^2 + 4x + 5 \neq 0.
\]

\[\forall x \in \mathbb{R}, x^2 + 4x + 5 > 0.
\]

a. (i) \( f \) is increasing on \( \mathbb{R} = (-\infty, \infty) \).
   (ii) \( f \) is never decreasing.

b. (i) \( f \) does not attain a local maximum.
   (ii) \( f \) does not attain a local minimum.
Let \( f(x) = (x^2 + 2x + 3)e^x \).

\( f''(x) = (2x + 4)e^x + (x^2 + 4x + 5)e^x \)
\[ = (x^2 + 6x + 9)e^x = (x + 3)^2 e^x \]

\( f'(x) = (x^2 + 4x + 5)e^x \)

**c.** (i) \( f \) is cc up on \((-\infty, \infty)\).

(ii) \( f \) is nowhere cc down.

**d.** \( f \) has no points of inflection.
0500-7. Let $f(x) = e^{4x-2x^2}$.

a. Find all critical numbers for $f$.

b. For each critical number for $f$, use the Second Derivative Test to determine whether, at that number, the function $f$ has a local maximum or a local minimum.

ANS: $f'(x) = e^{4x-x^2}(4-4x)$

$= -4(x-1)e^{4x-2x^2}$

$f''(x) = -4e^{4x-2x^2} - 4(x-1)e^{4x-2x^2}(4-4x)$

$= -4e^{4x-2x^2} + 4(x-1)e^{4x-2x^2}(4)(x-1)$

$= 4(-1 + 4(x-1)^2)e^{4x-2x^2}$
0500-7. Let \( f(x) = e^{4x-2x^2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), use the Second Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.

ANS: \( f'(x) = -4(x - 1)e^{4x-2x^2} \)

\( f''(x) = 4(-1 + 4(x - 1)^2)e^{4x-2x^2} \)

a. Critical numbers for \( f \): 1

b. \( f''(1) = 4(-1 + 4 \cdot 0^2)e^{4-2} < 0 \), so \( f \) has a local maximum at 1.
Let \( f(x) = x^3 e^{-x^2/2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), what does the Second Derivative Test tell you about that critical number?

c. For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
Let \( f(x) = x^3 e^{-x^2/2} \).

a. Find all critical numbers for \( f \).

**ANSWER:**

\[
\begin{align*}
\frac{d}{dx} f(x) &= (3x^2) e^{-x^2/2} + x^3 e^{-x^2/2} (-x) \\
&= (3x^2 - x^4) e^{-x^2/2} \\
&= -x^2 (x^2 - 3) e^{-x^2/2} \\
&= -(x + \sqrt{3}) x^2 (x - \sqrt{3}) e^{-x^2/2}
\end{align*}
\]

\[
\begin{align*}
\frac{d^2}{dx^2} f(x) &= (6x - 4x^3) e^{-x^2/2} + (3x^2 - x^4) e^{-x^2/2} (-x) \\
&= (6x - 7x^3 + x^5) e^{-x^2/2} \\
&= x (x^4 - 7x^2 + 6) e^{-x^2/2} \\
&= x (x^2 - 6) (x^2 - 1) e^{-x^2/2} \\
&= (x + \sqrt{6}) (x + 1) x (x - 1) (x - \sqrt{6}) e^{-x^2/2}
\end{align*}
\]
0500-8. Let \( f(x) = x^3 e^{-x^2/2} \).

a. Find all critical numbers for \( f \).

**ANSWER:** \( f'(x) = -(x + \sqrt{3})x^2(x - \sqrt{3})e^{-x^2/2} \)

a. critical numbers: \(-\sqrt{3}, 0, \sqrt{3}\)

\( f''(x) = (6x - 7x^3 + x^5)e^{-x^2/2} \)

\( = (x + \sqrt{6})(x + 1)x(x - 1)(x - \sqrt{6})e^{-x^2/2} \) \( f'' \) is odd.

\( p := \sqrt{3} + \sqrt{6} > 0, \quad r := \sqrt{3} - 1 > 0, \)

\( q := \sqrt{3} + 1 > 0, \quad s := \sqrt{3} - \sqrt{6} < 0. \)

b. \( f''(\sqrt{3}) = \sqrt{3}(pqrs)e^{-3/2} < 0 \)

Loc max at \( \sqrt{3} \), by the second derivative test.

\( f''(0) = 0 \)

At 0, the second derivative test gives NO information.

\( f''(-\sqrt{3}) = -f(\sqrt{3}) > 0 \)

Loc min at \(-\sqrt{3}\), by the second derivative test.
Let \( f(x) = x^3 e^{-x^2/2} \).

For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.

**ANSWER:** \( f'(x) = -(x + \sqrt{3})x^2(x - \sqrt{3})e^{-x^2/2} \)

Critical numbers: \( -\sqrt{3}, 0, \sqrt{3} \)

- \( f \) is decreasing on \( (-\infty, -\sqrt{3}] \).
- \( f \) is increasing on \([ -\sqrt{3}, \sqrt{3}] \).
- \( f \) is decreasing on \([ \sqrt{3}, \infty) \).

Then \( f \) has a local minimum at \( -\sqrt{3} \), nor a local maximum at 0 and \( f \) has a local maximum at \( \sqrt{3} \).
0500-9. Sketch the graph of a function $H : [0, 8] \rightarrow \mathbb{R}$ with the following properties:

- $H$ is continuous on $[0, 8]$;
- $H''$ is continuous on $(0, 8)$;
- $H(0) = 0$; $H(4) = H(8) = 2$;
- $H'(2) = H'(4) = H'(6) = 0$;
- $H'' < 0$ on $(0, 3)$;
- $H'' < 0$ on $(5, 8)$.

**ANSWER:**

There are many other answers.
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \) s.t. \( g \) attains a local min value of 20 at \(-1\) and a local max value of \(-16\) at 1.

**ANSWER:**

\[
g'(t) = 3at^2 + 2bt + c \\
0 = g'(-1) = 3a - 2b + c \\
0 = g'(1) = 3a + 2b + c \\
0 = [g'(1)] - [g'(-1)] = 4b \\
0 = b
\]

\[
0 = g'(1) = 3a + (2)(0) + c \\
-3a = c
\]
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \) s.t. \( g \) attains a local min value of 20 at \(-1\) and a local max value of \(-16\) at 1.

**ANSWER:** \( 0 = b \quad -3a = c \)

\[
g(t) = at^3 + bt^2 + ct + d
= at^3 - 3at + d
\]

\[
20 = g(-1) = -a + 3a + d = 2a + d
\]

\[
-16 = g(1) = a - 3a + d = -2a + d
\]

\[
4 = [g(1)] + [g(-1)] = 2d
\]

\[
2 = d
\]

\[
20 = g(-1) = 2a + d = 2a + 2
\]

\[
18 = 2a
\]

\[
9 = a
\]
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \) s.t. \( g \) attains a local min value of 20 at \(-1\) and a local max value of \(-16\) at \(1\).

**ANSWER:**

\[
\begin{align*}
0 &= b \\
-3a &= c \\
2 &= d \\
9 &= a \\
-27 &= -3a = c
\end{align*}
\]

\[
g(t) = at^3 + bt^2 + ct + d \\
= 9t^3 - 27t + 2
\]
0500-11. Let \( f(x) = \cot^3(x/2) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-11. Let \( f(x) = \cot^3(x/2) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

\[
 f(x) = \frac{\cos^3(x/2)}{\sin^3(x/2)}
\]

a. \( f \) is odd and \( 2\pi \)-periodic.

b. (i) \( \text{dom}[f] = \mathbb{R} \setminus \{\ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots\} \supseteq (0, \pi] \)

(ii) \( f(0) \) is undefined, so no \( y \)-intercept.

\( x \)-intercepts: \( f(x) = 0 \) iff \( \cos(x/2) = 0 \) iff \( x \in \{\ldots, -3\pi, -\pi, \pi, 3\pi, \ldots\} \)

\( f \) is pos. on \( \ldots \), \( (-2\pi, -\pi) \), \( (0, \pi) \), \( (2\pi, 3\pi) \), \ldots

\( f \) is neg. on \( \ldots \), \( (-3\pi, -2\pi) \), \( (-\pi, 0) \), \( (\pi, 2\pi) \), \( (3\pi, 4\pi) \), \ldots
Let $f(x) = \cot^3(x/2)$.

a. Describe the symmetries, if any, of $f$.

b. Find all max intervals of pos/neg for $f$.

Also:

(i) What is the domain of $f$?

(ii) Find all $x$- and $y$-intercepts of $f$.

(iii) Find all vert/horiz asymptotes of $f$.

**ANSWER:**

$$f(x) = \frac{\cos^3(x/2)}{\sin^3(x/2)}$$

$f$ is odd and $2\pi$-periodic.

b. (iii) $f$ is periodic and nonconstant, so $f$ has no horizontal asymptotes.

$$\lim_{x \to 0^-} f(x) = -\infty \quad \lim_{x \to 0^+} f(x) = \infty$$

Then $x = 0$ is a vertical asymptote for $f$.

So, as $f$ is $2\pi$-periodic, its vert. asymptotes are:

$$\ldots, x = -4\pi, x = -2\pi, \quad x = 0, \quad x = 2\pi, x = 4\pi, \ldots$$
0500-11. Let \( f(x) = \cot^3(x/2) \).

c. Find all max intervals of incr/decr for \( f \).

ANS: \( \text{dom}[f] = \mathbb{R} \setminus \{ \ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots \} \supseteq (0, \pi] \)

\[ f \text{ is odd and } 2\pi\text{-periodic.} \]

c. \( f'(x) = 3[\cot^2(x/2)][-\csc^2(x/2)][1/2] \]

\[ = -\frac{3\cos^2(x/2)}{2\sin^4(x/2)} \text{ is neg. on } 0 < x < \pi, \]

because \( \sin(x/2) \) and \( \cos(x/2) \) are both positive on \( 0 < x < \pi \).

\( f \) is decreasing on \( (0, \pi] \)

\( f \) odd gives: \( f \) decreasing on \([-\pi, 0) \)

\( f \) 2\( \pi \)-periodic gives: \( f \) decreasing on \([\pi, 2\pi) \)

Then \( f \) is decreasing on \((0, 2\pi)\).

\( f \) is 2\( \pi \)-periodic gives:

\( f \) decr on \((2n\pi, 2n\pi + 2\pi), \quad \forall n \in \mathbb{Z} \).
0500-11. Let \( f(x) = \cot^3(x/2) \).

**d. Find all max intervals of cc up/cc dn for \( f \).**

**ANS:**

\[
\text{dom}[f] = \mathbb{R} \setminus \{\ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots\} \supseteq (0, \pi] \\
\text{\( f \) is odd and } 2\pi\text{-periodic.}
\]

\[
f'(x) = 3[\cot^2(x/2)][-\csc^2(x/2)][1/2] \\
= -\frac{3 \cos^2(x/2)}{2 \sin^4(x/2)}
\]

**d.** \( f''(x) = \frac{6(\sin^5(x/2))(\cos(x/2)) + 12(\cos^3(x/2))(\sin^3(x/2))}{4 \sin^8(x/2)} \)

is positive on \( 0 < x < \pi \),

because \( \sin(x/2) \) and \( \cos(x/2) \) are both positive on \( 0 < x < \pi \).

\( f \) is concave up on \( (0, \pi] \)

\( f \) odd gives: \( f \) concave down on \( [-\pi, 0) \)
Let \( f(x) = \cot^3(x/2) \).

d. Find all max intervals of cc up/cc dn for \( f \).

ANS: \[ \text{dom}[f] = \mathbb{R} \setminus \{\ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots\} \supseteq (0, \pi] \]

\( f \) is odd and \( 2\pi \)-periodic.

d. \( f \) is concave up on \((0, \pi]\)

\( f \) odd gives: \( f \) concave down on \([-\pi, 0)\)

\( f \) is \( 2\pi \)-periodic gives:

\( f \) cc up on \((2n\pi, 2n\pi + \pi]\),
\( f \) cc dn on \([2n\pi - \pi, 2n\pi), \quad \forall n \in \mathbb{Z} \)
Let \( f(x) = \cot^3(x/2) \).

**e. Sketch the graph of \( f \).**

**ANS:**

**e.**
Let \( f(x) = \ln(9 - x^2) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = \ln(9 - x^2) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:** a. \( f \) is even.

b. (i) \( 9 - x^2 > 0 \iff -3 < x < 3 \).

\[ \text{dom}[f] = (-3, 3) \supseteq [0, 3) \]

(ii) \( f(0) = \ln 9 \) is the \( y \)-intercept.

\( x \)-intercepts:

\[ [9 - x^2 = 1] \iff [x = -2\sqrt{2} \text{ or } x = 2\sqrt{2}] \]

\[ [f(x) = 0] \iff [x = -2\sqrt{2} \text{ or } x = 2\sqrt{2}] \]
0500-12. Let \( f(x) = \ln(9 - x^2) \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:** \( f \) is even \( \text{dom}[f] = (-3, 3) \supseteq [0, 3) \)

b. \([f(x) = 0] \) iff \( [x = -2\sqrt{2} \text{ or } x = 2\sqrt{2}] \)

\[ 9 - x^2 < 1 \text{ on } 2\sqrt{2} < x < 3. \]
\[ 9 - x^2 > 1 \text{ on } -2\sqrt{2} < x < 2\sqrt{2}. \]
\[ 9 - x^2 < 1 \text{ on } -3 < x < -2\sqrt{2}. \]

\( f \) is negative on \((2\sqrt{2}, 3)\).
\( f \) is positive on \((-2\sqrt{2}, 2\sqrt{2})\).
\( f \) is negative on \((-3, -2\sqrt{2})\).
0500-12. Let \( f(x) = \ln(9 - x^2) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

**Answer:** \( f \) is even \( \quad \text{dom}[f] = (-3, 3) \supseteq [0, 3] \)

b. (iii) \( \text{dom}[f] = (-3, 3) \),
   
   so \( f \) has no horizontal asymptotes.

\[
\lim_{x \to -3^+} f(x) = -\infty = \lim_{x \to 3^-} f(x)
\]

\( x = -3 \) and \( x = 3 \) are the vertical asymptotes for \( f \).
0500-12. Let \( f(x) = \ln(9 - x^2) \).

**c. Find all max intervals of incr/decr for \( f \).**

**ANSWER:** \( f \) is even
\[ \text{dom}[f] = (-3, 3) \supseteq [0, 3) \]

\[ f'(x) = \frac{-2x}{9 - x^2} \] is neg. on \( 0 < x < 3 \).
\( f \) is decreasing on \([0, 3)\).

Even gives: increasing on \((-3, 0]\)

**d.** \( f''(x) = \frac{(9 - x^2)(-2) - (-2x)(-2x)}{(9 - x^2)^2} = \frac{-2x^2 - 18}{(9 - x^2)^2} \)
Let \( f(x) = \ln(9 - x^2) \).

**d. Find all max intervals of cc up/cc dn for \( f \).**

**ANSWER:** \( f \) is even \( \text{dom}[f] = (-3, 3) \supseteq [0, 3) \)

\[
f''(x) = \frac{-2x^2 - 18}{(9 - x^2)^2}
\]

is negative on \(-3 < x < 3\).

\( f \) is concave down on \((-3, 3)\).
0500-12. Let $f(x) = \ln(9 - x^2)$.

e. Sketch the graph of $f$.

ANSWER:

e.
0500-13. Let \( f(x) = \frac{2x}{\sqrt{9 - x^2}} \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).
d. Find all max intervals of cc up/cc dn for \( f \).
e. Sketch the graph of \( f \).
Let \( f(x) = \frac{2x}{\sqrt{9 - x^2}} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

\[ 9 - x^2 > 0 \iff -3 < x < 3 \]

\[ (i) \text{ dom}[f] = (-3, 3) \]

\[ (ii) f(0) = 0 \text{ is the } y \text{-intercept.} \]

\[ x \text{-intercepts: } [f(x) = 0] \iff [x = 0] \]

\( f \) is positive on \((0, 3)\) and negative on \((-3, 0)\).
Let \( f(x) = \frac{2x}{\sqrt{9 - x^2}} \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

\( f \) is odd. \quad \text{dom}[f] = (-3, 3)

b.(iii) vertical/horizontal asymptotes:

\[
f(x) = \frac{2x}{\sqrt{9 - x^2}} \quad x \geq 0 \quad \frac{2}{\sqrt{(9/x^2)} - 1} \quad x \to 3^-
\]

\[
f \text{ odd gives: } f(x) \to -\infty \quad x \to -3^+
\]

\( x = -3 \) and \( x = 3 \) are the vertical asymptotes of \( f \).

\[ \text{dom}[f] = (-3, 3), \text{ so } \]

\( f \) has no horizontal asymptotes.
0500-13. Let \( f(x) = \frac{2x}{\sqrt{9 - x^2}} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:** \( f \) is odd. \( \text{dom}[f] = (-3,3) \)

c. 

\[
f'(x) = \frac{([9 - x^2]^{1/2})(2) + (2x)([1/2][9 - x^2]^{-1/2}[+2x])}{9 - x^2}
\]

\[
= \frac{2[9 - x^2]^{1/2} + (2x^2)[9 - x^2]^{-1/2}}{9 - x^2}
\]

\[
= \frac{2[9 - x^2] + (2x^2)[1]}{[9 - x^2]^{3/2}}
\]

\[
= \frac{18}{[9 - x^2]^{3/2}}
\]
c. Find all max intervals of incr/decr for $f$. 

**ANSWER:** $f$ is odd.

$\text{dom}[f] = (-3, 3)$

c. $f'(x) = \frac{18}{[9 - x^2]^{3/2}}$ is positive on $(-3, 3)$.

$f$ is increasing on $(-3, 3)$. 
0500-13. Let \( f(x) = \frac{2x}{\sqrt{9-x^2}} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is odd. \( \text{dom}[f] = (-3, 3) \)

\[
f'(x) = \frac{18}{[9-x^2]^{3/2}} = 18[9-x^2]^{-3/2}
\]

d. \( f''(x) = 18(\frac{3}{2})[9-x^2]^{-5/2}[+2x] \)

\[
= \frac{54x}{[9-x^2]^{5/2}}
\]

is negative on \(-3 < x < 0\) and positive on \(0 < x < 3\).

\( f \) is concave down on \((-3, 0]\) and concave up on \([0, 3)\).
0500-13. Let \( f(x) = \frac{2x}{\sqrt{9 - x^2}} \).

e. Sketch the graph of \( f \).

**ANSWER:**
e. 

![Graph of f(x)](image)
0500-14. Let \( f(x) = x^5 + 5x^3 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-14. Let \( f(x) = x^5 + 5x^3 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

a. \( f \) is odd.

b. (i) \( \text{dom}[f] = \mathbb{R} \supseteq [0, \infty) \)
   \[
f(x) = (x^2 + 5)x^3
   \]
   (ii) \( f(0) = 0 \) is the \( y \)-intercept.
   \( x \)-intercepts: \([f(x) = 0] \) iff \([x = 0]\)
   \[
f(x) \text{ is negative on } x < 0
   \]
   and positive on \( 0 < x \).
   (iii) vertical/horizontal asymptotes: none
0500-14. Let \( f(x) = x^5 + 5x^3 \).

**c. Find all max intervals of incr/decr for \( f \).**

**ANSWER:** \( f \) is odd. \( \text{dom}[f] = \mathbb{R} \supseteq [0, \infty) \)

\[ f'(x) = 5x^4 + 15x^2 \]
\[ = 5(x^2 + 3)x^2 \]

is positive, for all \( x \in \mathbb{R} = (-\infty, \infty) \).

\( f \) is increasing on \( \mathbb{R} = (-\infty, \infty) \).

Also, \( f'(0) = 0 \).
0500-14. Let \( f(x) = x^5 + 5x^3 \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is odd.

\[
f'(x) = 5x^4 + 15x^2
\]

\[
f''(x) = 20x^3 + 30x
\]

\[
= 20 \left( x^2 + \frac{3}{2} \right) x
\]

is negative on \( x < 0 \)

and positive on \( 0 < x \).

\( f \) is concave down on \( (-\infty, 0] \)

and concave up on \( [0, \infty) \).
Let \( f(x) = x^5 + 5x^3 \).

e. Sketch the graph of \( f \).

**ANSWER:**

e.
Let \( f(x) = \frac{1}{x^3 - 1} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

   Also:
   
   (i) What is the domain of \( f \)?
   
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-15. Let \( f(x) = \frac{1}{x^3 - 1} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

**ANSWER:**

a. **NO** symmetries.

b. \( x^3 - 1 > 0 \) on \( x > 1 \)
\( x^3 - 1 = 0 \) at \( x = 1 \)
\( x^3 - 1 < 0 \) on \( x < 1 \)

(i) \( \text{dom}[f] = \mathbb{R} \setminus \{1\} \)

(ii) \( f(0) = -1 \) is the \( y \)-intercept.
\( \forall x \in \text{dom}[f], \ f(x) \neq 0, \) so no \( x \)-intercepts

\[ f(x) = \frac{1}{x^3 - 1} \] is negative on \( x < 1 \)
and positive on \( 1 < x \).
0500-15. Let \( f(x) = \frac{1}{x^3 - 1} \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**
\[
\text{\( \text{dom}[f] = \mathbb{R}\setminus\{1\} \)}
\]

(iii) \( \lim_{x \to \infty} f(x) = 0 \) and \( \lim_{x \to -\infty} f(x) = 0 \)

\( y = 0 \) is the only horizontal asymptote.

- \( x^3 - 1 > 0 \) on \( x > 1 \)
- \( x^3 - 1 = 0 \) at \( x = 1 \)
- \( x^3 - 1 < 0 \) on \( x < 1 \)

\( \lim_{x \to 1^-} f(x) = -\infty \) and \( \lim_{x \to 1^+} f(x) = \infty \)

\( x = 1 \) is the only vertical asymptote.
0500-15. Let \( f(x) = \frac{1}{x^3 - 1} \).

**c.** Find all max intervals of incr/decr for \( f \).  

**ANSWER:**

\[ f(x) = \left[ x^3 - 1 \right]^{-1} \]

\[ f'(x) = -\left[ x^3 - 1 \right]^{-2} \left[ 3x^2 \right] = \frac{-3x^2}{(x^3 - 1)^2} \]

is negative on \( x \in \mathbb{R} \setminus \{0, 1\} \).

\( f \) is decreasing on \((-\infty, 1)\) and on \((1, \infty)\).

Also, \( f'(0) = 0 \).
Let \( f(x) = \frac{1}{x^3 - 1} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[
f'(x) = \frac{-3x^2}{(x^3 - 1)^2}
\]

\[
f''(x) = \frac{[\frac{8(8x^3 - 1)}{(x^3 - 1)^4}] [-6x] - [\frac{-3x^2}{(x^3 - 1)^2}] [2(3x^2 - 1)(3x^2)]}{(x^3 - 1)^4}
\]

\[
= \frac{[x^3 - 1] [-6x] - [\frac{-3x^2}{(x^3 - 1)^2}] [6x^2]}{(x^3 - 1)^3}
\]

\[
\text{dom}[f] = \mathbb{R} \setminus \{1\}
\]
Let \( f(x) = \frac{1}{x^3 - 1} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[
\text{dom}[f] = \mathbb{R} \setminus \{1\}
\]

\[
d. \ f''(x) = \frac{\left[ x^3 - 1 \right] [-6x] - \left[ -3x^2 \right][6x^2]}{(x^3 - 1)^3}
\]

\[
= \frac{-6x^4 + 6x}{(x^3 - 1)^3} + \frac{18x^4}{(x^3 - 1)^3}
\]

\[
= \frac{12x^4 + 6x}{(x^3 - 1)^3} = \frac{12x(x^3 + \frac{1}{2})}{(x^3 - 1)^3}
\]
0500-15. Let \( f(x) = \frac{1}{x^3 - 1} \).

**ANSWER:**

**d. Find all max intervals of cc up/cc dn for \( f \).**

\[
\text{dom}[f] = \mathbb{R}\{1\}
\]

\[
(x^3 - 1)^3 > 0 \text{ on } x > 1
\]

\[
(x^3 - 1)^3 < 0 \text{ on } x < 1
\]

\[
x^3 + \frac{1}{2} > 0 \text{ on } x > -1/\sqrt[3]{2}
\]

\[
x^3 + \frac{1}{2} < 0 \text{ on } x < -1/\sqrt[3]{2}
\]

\[f''(x) = \frac{12x(x^3 + \frac{1}{2})}{(x^3 - 1)^3}\]

is negative on \( x < -1/\sqrt[3]{2} \)

and positive on \(-1/\sqrt[3]{2} < x < 0\)

and negative on \(0 < x < 1\)

and positive on \(1 < x\).

\(f\) is concave down on \((-\infty, -1/\sqrt[3]{2}]\)

and concave up on \([-1/\sqrt[3]{2}, 0]\)

and concave down on \([0, 1)\)

and concave up on \((1, \infty)\).
Let $f(x) = \frac{1}{x^3 - 1}$.

e. Sketch the graph of $f$.

**ANSWER:**

e. $-1/\sqrt[3]{2} \approx -0.79$

\[\begin{array}{c}
-1/\sqrt[3]{2} \approx -0.79 \\
-2/3 \\
-1 \\
1
\end{array}\]
0500-16. Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

**a.** Describe the symmetries, if any, of \( f \).

**b.** Find all max intervals of pos/neg for \( f \). Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

**c.** Find all max intervals of incr/decr for \( f \).

**d.** Find all max intervals of cc up/cc dn for \( f \).

**e.** Sketch the graph of \( f \).
NEW 0500-16. Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

**ANSWER:**

a. **NO** symmetries

b. \( 4^2 - 4 \cdot 1 \cdot 7 < 0 \), so, \( \forall x \in \mathbb{R}, x^2 + 4x + 7 \neq 0 \)

\( \forall x \in \mathbb{R}, x^2 + 4x + 7 > 0 \)

(i) \( \text{dom}[f] = \mathbb{R} = (-\infty, \infty) \)

(ii) \( f(0) = \sqrt{7} \) is the \( y \)-intercept.

\( \forall x \in \text{dom}[f], f(x) \neq 0 \), so no \( x \)-intercepts

\( f \) is positive on \( \mathbb{R} = (-\infty, \infty) \).

Note: \( f \) is symmetric about \( x = -2 \), but this is **not** one of our standard symmetries.
Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

b. Find all max intervals of pos/neg for \( f \).

Also:
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

\[
\text{dom}[f] = \mathbb{R} = (-\infty, \infty)
\]
\[
\forall x \in \mathbb{R}, \ x^2 + 4x + 7 > 0
\]

b. (iii) vertical/horizontal asymptotes: none

**Note:** \( f(x) - x \to 2 \), as \( x \to \infty \),

so \( y = x + 2 \) is a slant asymptote for \( f \).

**Note:** \( f(x) + x \to -2 \), as \( x \to -\infty \),

so \( y = -x - 2 \) is a slant asymptote for \( f \).
Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

**c. Find all max intervals of incr/decr for \( f \).**

**ANSWER:**

\[
\text{dom}[f] = \mathbb{R} = (-\infty, \infty) \\
\forall x \in \mathbb{R}, \quad x^2 + 4x + 7 > 0
\]

\[
f(x) = (x^2 + 4x + 7)^{1/2}
\]

\[
f'(x) = \frac{1}{2} (x^2 + 4x + 7)^{-1/2} (2x + 4)
\]

\[
= \frac{x + 2}{\sqrt{x^2 + 4x + 7}}
\]

is negative on \( x < -2 \)
and positive on \( -2 < x \).

\( f \) is decreasing on \( (-\infty, -2] \)
and increasing on \( [-2, \infty) \).
0500-16. Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[ \text{dom}[f] = \mathbb{R} = (-\infty, \infty) \]
\[ \forall x \in \mathbb{R}, \ x^2 + 4x + 7 > 0 \]

\[ f'(x) = \frac{x + 2}{\sqrt{x^2 + 4x + 7}} = \frac{x + 2}{(x^2 + 4x + 7)^{1/2}} \]

d. \( f''(x) = \)

\[ \left( \frac{1}{2} \right) \frac{[1] - [x + 2] [1/2] (x^2 + 4x + 7)^{-1/2} (2x + 4)}{x^2 + 4x + 7} \]

\[ = \frac{[x^2 + 4x + 7] [1] - [x + 2] [(1/2)(2x + 4)]}{(x^2 + 4x + 7)^{3/2}} \]
0500-16. Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[
d. \ f''(x) = \\
\frac{\left[ x^2 + 4x + 7 \right] [1] - [x + 2] \left[ (1/2)(2x + 4) \right]}{\left( x^2 + 4x + 7 \right)^{3/2}} \\
= \frac{\left[ x^2 + 4x + 7 \right] - [x + 2][x + 2]}{\left( x^2 + 4x + 7 \right)^{3/2}} \\
= \frac{\left[ x^2 + 4x + 7 \right] - [x^2 + 4x + 4]}{\left( x^2 + 4x + 7 \right)^{3/2}}
\]

\( \text{dom}[f] = \mathbb{R} = (-\infty, \infty) \)

\( \forall x \in \mathbb{R}, \ x^2 + 4x + 7 > 0 \)
0500-16. Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

**ANSWER:**

\[ \text{dom}[f] = \mathbb{R} = (-\infty, \infty) \]

\[ \forall x \in \mathbb{R}, \quad x^2 + 4x + 7 > 0 \]

\[ f''(x) = \frac{\left[ x^2 + 4x + 7 \right] - \left[ x^2 + 4x + 4 \right]}{(x^2 + 4x + 7)^{3/2}} \]

\[ = \frac{3}{(x^2 + 4x + 7)^{3/2}} \]

is positive, for all \( x \in \mathbb{R} \).

\( f \) is concave up on \( \mathbb{R} = (-\infty, \infty) \).
Let \( f(x) = \sqrt{x^2 + 4x + 7} \).

e. Sketch the graph of \( f \).

\text{ANSWER: e.}
0500-17. Let \( f(x) = 2x + 1 - \cos x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
NEW 0500-17. Let $f(x) = 2x + 1 - \cos x$.

a. Describe the symmetries, if any, of $f$.

b. Find all max intervals of pos/neg for $f$.

Also:

(i) What is the domain of $f$?

(ii) Find all $x$- and $y$-intercepts of $f$.

(iii) Find all vert/horiz asymptotes of $f$.

ANSWER: Note: $f(x + 2\pi) = (f(x)) + 4\pi$

a. NO symmetries.

b. (i) $\text{dom}[f] = \mathbb{R} = (-\infty, \infty)$

(ii) deferred until after c.

max intervals of pos/neg for $f$

also deferred until after c.

(iii) vertical/horizontal asymptotes: none

Note: $f$ is symm. about $(-\pi/2, 1 - \pi)$, but this is not one of our standard symmetries.
0500-17. Let \( f(x) = 2x + 1 - \cos x \).

**c. Find all max intervals of incr/decr for \( f \).**

**ANSWER:**

\[
dom[f] = \mathbb{R} = (-\infty, \infty)
\]

**c.** \( f'(x) = 2 + \sin x \)

is positive, \( \forall x \in \mathbb{R} \)

\( f \) is increasing on \( \mathbb{R} \).

**b. (ii)** \( f(0) = 0 \) is the \( y \)-intercept.

So, because \( f \) is increasing on \( \mathbb{R} \),

\( f \) is positive on \( (0, \infty) \)

and \( f \) is negative on \( (-\infty, 0) \).

\( x \)-intercepts: \( [f(x) = 0] \) iff \( [x = 0] \)
Let \( f(x) = 2x + 1 - \cos x \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[
f'(x) = 2 + \sin x
\]

\[
f''(x) = \cos x
\]

is neg. on \((2n + \frac{1}{2})\pi < x < (2n + \frac{3}{2})\pi\), \( \forall n \in \mathbb{Z} \), and pos. on \((2n - \frac{1}{2})\pi < x < (2n + \frac{1}{2})\pi\), \( \forall n \in \mathbb{Z} \).

\( f \) is cc down on \([(2n + \frac{1}{2})\pi, (2n + \frac{3}{2})\pi]\), \( \forall n \in \mathbb{Z} \), and cc up on \([(2n - \frac{1}{2})\pi, (2n + \frac{1}{2})\pi]\), \( \forall n \in \mathbb{Z} \).
0500-17. Let $f(x) = 2x + 1 - \cos x$.

e. Sketch the graph of $f$.

**ANSWER:**

e.
0500-18. Let \( f(x) = -2x^2e^{-x^2/2} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-18. Let \( f(x) = -2x^2e^{-x^2/2} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

a. \( f \) is even.

b. (i) \( \text{dom}[f] = \mathbb{R} \supseteq [0, \infty) \)

(ii) \( f(0) = 0 \) is the \( y \)-intercept.

\( x \)-intercepts: \([f(x) = 0] \text{ iff } [x = 0] \)

\( f \) is negative on \((0, \infty)\).

\( f \) even gives: \( f \) is negative on \((-\infty, 0)\).

(iii) vertical asymptotes: none

horizontal asymptotes: \( y = 0 \)
$0500-18$. Let $f(x) = -2x^2e^{-x^2/2}$.

**c. Find all max intervals of incr/decr for $f$.**

**ANSWER:**

**c.** $f'(x) = \left[-4x\right]\left[e^{-x^2/2}\right] + \left[-2x^2\right]\left[e^{-x^2/2}(-x)\right]$

$$= \left[2x^3 - 4x\right]\left[e^{-x^2/2}\right] = 2x[x^2 - 2]\left[e^{-x^2/2}\right]$$

$$= 2[x + \sqrt{2}]x[x - \sqrt{2}]\left[e^{-x^2/2}\right]$$

is negative on $x < -\sqrt{2}$,
and positive on $-\sqrt{2} < x < 0$,
and negative on $0 < x < \sqrt{2}$,
and positive on $\sqrt{2} < x$.

$f$ is decreasing on $(-\infty, -\sqrt{2}]$,
and increasing on $[-\sqrt{2}, 0]$,
and decreasing on $[0, \sqrt{2}]$,
and increasing on $[\sqrt{2}, \infty)$. 
0500-18. Let \( f(x) = -2x^2 e^{-x^2/2} \).

\( \text{NEW} \)

\( \text{d. Find all max intervals of cc up/cc dn for } f. \)

**ANSWER:** \( f'(x) = [2x^3 - 4x] \left[ e^{-x^2/2} \right] \)

\( \text{d. } f''(x) = [6x^2 - 4] \left[ e^{-x^2/2} \right] + [2x^3 - 4x] \left[ e^{-x^2/2} (-x) \right] \)

\( = [6x^2 - 4 - 2x^4 + 4x^2] \left[ e^{-x^2/2} \right] \)

\( = [-2x^4 + 10x^2 - 4] \left[ e^{-x^2/2} \right] \)

\( = -2[x^4 - 5x^2 + 2] \left[ e^{-x^2/2} \right] \)

\( = -2 \left[ x^2 - a^2 \right] \left[ x^2 - b^2 \right] \left[ e^{-x^2/2} \right] \)

\( = -2 \left[ x + b \right] \left[ x + a \right] \left[ x - a \right] \left[ x - b \right] \left[ e^{-x^2/2} \right] \)

\( a := \sqrt{\frac{5 - \sqrt{17}}{2}} \)

\( b := \sqrt{\frac{5 + \sqrt{17}}{2}} \)

\( z^2 - 5z + 2 = (z - a^2)(z - b^2) \)

\( z \mapsto x^2 \)
Let \( f(x) = -2x^2e^{-x^2/2} \).

**d.** Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( a := \sqrt{\frac{5 - \sqrt{17}}{2}} \quad b := \sqrt{\frac{5 + \sqrt{17}}{2}} \)

\( f''(x) = -2 [x + b] [x + a] [x - a] [x - b] e^{-x^2/2} \)

is negative on \( x < -a \),
and positive on \( -a < x < -b \),
and negative on \( -b < x < b \),
and positive on \( b < x < a \),
and negative on \( a < x \).

\( f \) is concave down on \( (-\infty, -a] \),
and concave up on \( [-a, -b] \),
and concave down on \( [-b, b] \),
and concave up on \( [b, a] \),
and concave down on \( [a, \infty) \).
0500-18. Let \( f(x) = -2x^2e^{-x^2/2} \).

**e. Sketch the graph of \( f \).**

**ANSWER:**

\[
\begin{align*}
  a & := \sqrt{\frac{5 - \sqrt{17}}{2}} \\
  & \doteq 0.66 \\

  b & := \sqrt{\frac{5 + \sqrt{17}}{2}} \\
  & \doteq 2.14
\end{align*}
\]

\[
\begin{align*}
  f(a) & \doteq -0.70 \\
  f(b) & \doteq -0.93
\end{align*}
\]

\[
\begin{align*}
  f(a) = f(-a) \\
  f(b) = f(-b)
\end{align*}
\]

\[
\begin{align*}
  f(\sqrt{2}) & = f(-\sqrt{2}) \\
  \sqrt{2} & \doteq 1.41 \\
  f(\sqrt{2}) & \doteq -1.47
\end{align*}
\]
0500-19. Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-19. Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

**ANSWER:** \( f(x) = \frac{x^2 - 6x - 7}{x - 1} = \frac{(x + 1)(x - 7)}{x - 1} \)

a. **NO** symmetries

b. (i) \( \text{dom}[f] = \mathbb{R} \setminus \{1\} \)

(ii) \( f(0) = 7 \) is the \( y \)-intercept.

\( x \)-intercepts: \( [f(x) = 0] \iff [x = -1 \text{ or } x = 7] \)

- \( f(x) \) is negative on \( x < -1 \),
- positive on \( -1 < x < 1 \),
- negative on \( 1 < x < 7 \) and
- positive on \( 7 < x \).
0500-19. Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:** \( \text{dom}[f] = \mathbb{R}\setminus\{1\} \)

\( f(x) \) is negative on \( x < -1 \),
positive on \( -1 < x < 1 \),
negative on \( 1 < x < 7 \)
and positive on \( 7 < x \).

b. (iii) \( f \) has no horizontal asymptotes.

\[ \lim_{x \to 1^-} f(x) = \infty \quad \lim_{x \to 1^+} f(x) = -\infty \]

\( x = 1 \) is the only vertical asymptote for \( f \).

\[ \lim_{x \to \pm\infty} [(f(x)) - x] = -5, \]
so \( f(x) \) is slant asymptotic to \( x - 5 \), as \( x \to \pm\infty \).
Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

\[ f'(x) = \frac{[x - 1][2x - 6] - [x^2 - 6x - 7][1]}{(x - 1)^2} \]

\[ = \frac{[2x^2 - 8x + 6] - [x^2 - 6x - 7]}{(x - 1)^2} \]

\[ = \frac{x^2 - 2x + 13}{(x - 1)^2} \]

\[ \text{dom}[f] = \mathbb{R} \setminus \{1\} \]
Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

\[
\text{dom}[f] = \mathbb{R} \setminus \{1\}
\]

\[
f'(x) = \frac{x^2 - 2x + 13}{(x - 1)^2}
\]

\((-2)^2 - 4 \cdot 1 \cdot 13 < 0, \text{ so }
\forall x \in \mathbb{R}, x^2 - 2x + 13 \neq 0.
\]

\(\forall x \in \mathbb{R}, x^2 - 2x + 13 > 0.
\]

is positive on \( x < 1 \)

and positive on \( 1 < x \).

\( f \) is increasing on \((-\infty, 1)\)

and increasing on \((1, \infty)\).
0500-19. Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

**d. Find all max intervals of cc up/cc dn for \( f \).**

**ANSWER:**

\[ \text{dom}[f] = \mathbb{R} \setminus \{1\} \]

\[ f'(x) = \frac{x^2 - 2x + 13}{(x - 1)^2} \]

\[ f''(x) = \frac{[(x - 1)^2][2x - 2] - [x^2 - 2x + 13][2(x - 1)(1)]}{(x - 1)^3} \]

\[ = \frac{[x - 1][2x - 2] - [x^2 - 2x + 13][2]}{(x - 1)^3} \]

\[ = \frac{2x^2 - 4x + 2 - [2x^2 - 4x + 26]}{(x - 1)^3} \]

\[ = \frac{-24}{(x - 1)^3} \]
Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

**d. Find all max intervals of cc up/cc dn for \( f \).**

**ANSWER:**

\[
dom[f] = \mathbb{R} \setminus \{1\}
\]

\[
f'(x) = \frac{x^2 - 2x + 13}{(x - 1)^2}
\]

\[
f''(x) = \frac{-24}{(x - 1)^3}
\]

is positive on \( x < 1 \) and negative on \( 1 < x \).

\( f \) is concave up on \( (-\infty, 1) \) and concave down on \( (1, \infty) \).
0500-19. Let \( f(x) = \frac{x^2 - 6x - 7}{x - 1} \).

e. Sketch the graph of \( f \).

ANSWER: