CALCULUS
Antidifferentiation problems
NEW
0560-1. Find all antiderivatives in $x$ of

$$\sqrt{2}x^3 - x^2 + ex.$$

**ANSWER:**

all antiderivatives: $$\frac{\sqrt{2}x^4}{4} - \frac{x^3}{3} + \frac{ex^2}{2} + C$$

0560-2. Find all antiderivatives in $t$ of

$$\left(\pi \sqrt[5]{t} - 2 \sqrt[3]{t}\right) t.$$

**ANSWER:**

$$\left(\pi \sqrt[5]{t} - 2 \sqrt[3]{t}\right) t = \pi t^{6/5} - 2t^{4/3}$$

all antiderivatives: $$\frac{\pi t^{11/5}}{11/5} - \frac{2t^{7/3}}{7/3} + C$$
0560-3. Find all antiderivatives in \( t \) of
\[
\frac{\sqrt[6]{t} + 8 \sqrt[7]{t}}{\sqrt[5]{t}}.
\]

**Answer:**
\[
\frac{\sqrt[6]{t} + 8 \sqrt[7]{t}}{\sqrt[5]{t}} = t^{-1/30} + 8t^{-2/35}
\]

All antiderivatives:
\[
\frac{t^{29/30}}{29/30} + \frac{8t^{33/35}}{33/35} + C
\]

0560-4. Find all antiderivatives in \( s \) of
\[
\frac{7e^s + \cos s}{3}.
\]

**Answer:** All antiderivatives:
\[
\frac{7e^s + \sin s}{3} + C
\]
0560-5. Find the unique $f(x)$ such that

$$f'(x) = 5x^4 + 9x^2 - 6x \quad \text{and} \quad f(0) = 2.$$ 

**ANSWER:**

$$f(x) = x^5 + 3x^3 - 3x^2 + 2$$

0560-6. Find the unique $f(x)$ such that

$$f'(x) = \frac{3x^2 + 4}{x^{8\sqrt{x}}} \quad \text{and} \quad f(1) = 0.$$ 

**ANSWER:**

$$f'(x) = \frac{3x^2 + 4}{x^{8\sqrt{x}}} = 3x^{7/8} + 4x^{-9/8}$$

$$f(x) = \frac{3x^{15/8}}{15/8} + \frac{4x^{-1/8}}{-1/8} - \frac{3}{15/8} - \frac{4}{-1/8}$$
0560-7. Find the unique \( h(t) \) such that
\[
    h'(t) = 2 \sin t + 7 \cos t \quad \text{and} \quad h(0) = 4.
\]

**ANSWER:**
\[
    h(t) = -2 \cos t + 7 \sin t + 6
\]

0560-8. Find the unique \( p(t) \) such that
\[
    p''(t) = -2e^t + 4t^3, \quad p'(0) = -4 \quad \text{and} \quad p(0) = \sqrt{2}.
\]

**ANSWER:**
\[
    p'(t) = -2e^t + t^4 - 2
\]

\[
    p(t) = -2e^t + \frac{t^5}{5} - 2t + 2 + \sqrt{2}
\]
The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?
0560-9. The graph of \( f \) is shown below.

Which of the following could be the graph of an antiderivative of \( f \)?

**Answer:**
The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?
The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?

**ANSWER:**
The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?
The graph of \( f \) is shown below.

**Answer:**
The graph of $f$ is shown below.

Which of the following could be the graph of an antiderivative of $f$?
The graph of $f$ is shown below.

**ANSWER:**
A particle travels on a number line. Suppose its acceleration at time $t$ is $6t^2 - 2t + 3$, its position at time 0 is 4 and its velocity at time 0 is $-5$. Find an expression for its position at time $t$.

**ANSWER:**

velocity at time $t$: $2t^3 - t^2 + 3t - 5$

position at time $t$: $\frac{t^4}{2} - \frac{t^3}{3} + \frac{3t^2}{2} - 5t + 4$
We drop a heavy ball out of a window in a tall building. Its speed at the moment of impact with the ground is 144 feet per second. From what height was it dropped?

**ANSWER:**

- acceleration is: $32 \text{ ft/sec}^2$
- velocity $t$ seconds after release is: $32t \text{ ft/sec}$
- distance fallen in the first $t$ seconds after release is: $16t^2 \text{ ft}$

Let $t_0$ be the number of seconds between release and impact.

$32t_0 = 144$, so $t_0 = 4.5$ seconds.

Distance from release point to the ground is $16t_0^2 = 16 \cdot 4.5^2 = 324 \text{ ft.}$