CALCULUS
Area between curves:
Problems
NEW
Compute the shaded area shown in the picture below.

**Answer:**

\[
\int_{-1}^{2} 2x - (2x^2 - 4) \, dx \\
= \int_{-1}^{2} 4 + 2x - 2x^2 \, dx \\
= \left[ 4x + x^2 - \frac{2x^3}{3} \right]_{x=-1}^{x=2} \\
= \left[ 8 + 4 - \frac{16}{3} \right] - \left[ -4 + 1 + \frac{2}{3} \right] \\
= 12 - \frac{16}{3} + 3 - \frac{2}{3} = 15 - \frac{18}{3} \\
= 15 - 6 = 9
\]
0690-2. Let $R$ be the region enclosed inside $y = e^{-x}$, $y = x^2$, $x = -0.5$ and $x = 0.5$.

(a) Sketch the region $R$.

(b) Compute the area of the region $R$.

ANS:

(a)

(b) $\int_{-1/2}^{1/2} e^{-x} - x^2 \, dx$

\[ = \left[ \frac{e^{-x}}{-1} - \frac{x^3}{3} \right]_{x:-1/2}^{1/2} \]

\[ = \left[ \frac{e^{-1/2}}{-1} - \frac{1}{24} \right] - \left[ \frac{e^{1/2}}{-1} - \frac{-1/8}{3} \right] \]

\[ = -e^{-1/2} - \frac{1}{24} + e^{1/2} - \frac{1}{24} \]

\[ = e^{1/2} - e^{-1/2} - \frac{1}{12} \]
0690-3. Let \( R \) be the region enclosed inside 
\[ y = 2 \sin(\pi x / 6), \quad y = x \quad \text{and} \quad x \geq 0. \]

(a) Sketch the region \( R \).

(b) Compute the area of the region \( R \).

**ANSWER:**

(a) 
\[ y = 2 \sin(\pi x / 6) \]

(b) 
\[
\int_0^1 [2 \sin(\pi x / 6)] - x \, dx \\
= 2 \left[ \int_0^1 \sin(\pi x / 6) \, dx \right] - \left[ \int_0^1 x \, dx \right] \\
= 2 \left[ \frac{- \cos(\pi x / 6)}{\pi / 6} \right]_{x: \to 1} - \left[ \frac{x^2}{2} \right]_{x: \to 1} \\
= 2 \left[ \frac{- \sqrt{3}/2}{\pi / 6} - \frac{-1}{\pi / 6} \right] - \left[ \frac{1}{2} - 0 \right] \\
= 2 \left[ \frac{-3\sqrt{3}}{\pi} + \frac{6}{\pi} \right] - \frac{1}{2} \\
= 6 - \frac{1}{2} \\
= \frac{11}{2}
\]
Let \( R \) be the region enclosed inside \( y = 2 \sin(\pi x/6), \quad y = x \) and \( x \geq 0 \).

a. Sketch the region \( R \).

b. Compute the area of the region \( R \).

**ANSWER:**

\[
\begin{align*}
\text{b.} & \quad \int_0^1 [2\sin(\pi x/6)] - x \, dx \\
& = 2 \left[ \frac{-3\sqrt{3}}{\pi} + \frac{6}{\pi} \right] - \frac{1}{2} \\
& = 2 \left[ \frac{6}{\pi} - \frac{3\sqrt{3}}{\pi} \right] - \frac{1}{2} \\
& = \frac{12 - 6\sqrt{3}}{\pi} - \frac{1}{2} \\
& = \frac{24 - 12\sqrt{3} - \pi}{2\pi}
\end{align*}
\]
0690-4. Let $R$ be the region enclosed inside

$$y = x^2$$ and $$y = 2x + 8.$$ 

a. Sketch the region $R$.

b. Compute the area of the region $R$.

**ANSWER:**

a. 

b. 

$$
\int_{-2}^{4} (2x + 8) - x^2 \, dx \\
= \left[ (x^2 + 8x) - \frac{x^3}{3} \right]_{x=-2}^{x=4} \\
= \left[ (16 + 32) - \frac{64}{3} \right] - \left[ (4 - 16) - \left( \frac{-8}{3} \right) \right] \\
= 48 - \frac{64}{3} - (-12) + \left( \frac{-8}{3} \right) \\
= 60 - \frac{72}{3} \\
= 60 - 24 \\
= 36
$$
Let \( f(x) = e^{-x^2/10} \) and let \( g(x) = -x \). Estimate the area of the region bounded by \( y = f(x), \ y = g(x), \ x = 2 \) and \( x = 8 \) by computing \( R_3 S_2^8 (f - g) \).

**ANSWER:** Let \( h = f - g \),

so \( h(x) = e^{-x^2/10} - (-x) = e^{-x^2/10} + x \).

\[
R_3 S_2^8 (f - g) = R_3 S_2^8 h \\
= [2][h(4)] + [2][h(6)] + [2][h(8)] \\
= 2 \left[ e^{-16/10} + 4 \right] + \\
2 \left[ e^{-36/10} + 6 \right] + 2 \left[ e^{-64/10} + 8 \right] \\
= 2e^{-1.6} + 2e^{-3.6} + 2e^{-6.4} + 36
\]