CALCULUS
The limit game and
the exact definition of a limit
OLD
0150-1. For the function $f$ graphed below, what is the largest number $\delta$ such that

$$0 < |x - 2| < \delta \quad \Rightarrow \quad |(f(x)) - 1| < 0.3$$

?
0150-2. Let \( f(x) = 2x - 1 \).

Show a graph of \( y = f(x) \) that includes the points (2, 3), (3, 5) and (4, 7).

Find the largest number \( \delta \) such that
\[
|x - 3| < \delta \quad \Rightarrow \quad |(f(x)) - 5| < 1.
\]

0150-3. Let \( g(x) = [2x - 1]\left[\frac{x - 3}{x - 3}\right] \).

Show a graph of \( y = g(x) \) that includes the points (2, 3) and (4, 7).

Find the largest number \( \delta \) such that
\[
0 < |x - 3| < \delta \quad \Rightarrow \quad |(g(x)) - 5| < 1.
\]
In shop class, you are asked to build a square sheet of metal of area 100 square inches.

The area can be slightly off, but must be between 99 and 101 square inches.

Say you have access to a machine that will punch out a perfect square, and the side length (in inches) is controlled by a dial.

How close to 10 must you set the dial to get the area to be in the specified range? Give your answer to five decimal places.
Prove that \( \lim_{x \to 3} 7x = 21 \).

Your writeup should read:

Given \( \varepsilon > 0 \).

Let \( \delta = \ldots \).

Assume \( 0 < |x - 3| < \delta \).

Then \( |7x - 21| < 7\delta \).

Then \( |7x - 21| < \varepsilon \).

All you need do is fill in the ellipsis (\( \ldots \)) with a carefully chosen expression of \( \varepsilon \).

**Hint:** The last sentence in the writeup clearly follows from the penultimate sentence if \( 7\delta = \varepsilon \).