CALCULUS
Even more graphing problems
OLD
0500-1. Let \( f : [0, 15) \setminus \{8\} \to \mathbb{R} \) be as shown.

a. Find the maximal intervals on which

(i) \( f \) is increasing;
(ii) \( f \) is decreasing;
(iii) \( f \) is concave up;
and (iv) \( f \) is concave down.

b. Find all points of inflection for \( f \).
0500-2. Let \( f : [0, 15) \setminus \{8\} \to \mathbb{R} \) be continuous from the right at 0. The graph of \( f' \) is shown below.

**Find** the maximal intervals on which

(i) \( f \) is concave up;

and (ii) \( f \) is concave down.
0500-3. Let \( f : [0, 14] \rightarrow \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.
0500-4. Let $f$ be continuous on $[0, 14]$.
The graph of $f'$ is shown below.

a. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.

b. At what numbers does $f$ have
   (i) a local maximum?
   (ii) a local minimum?
0500-5. Let \( f(x) = x^4 - 4x^3 + 4x^2 + 9 \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.
Let \( f(x) = (x^2 + 1)e^{-x} \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

c. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.

d. Find all points of inflection for \( f \).
0500-7. Let $f(x) = xe^{-x^2/2}$.

a. Find all critical numbers for $f$.

b. For each critical number for $f$, use the Second Derivative Test to determine whether, at that number, the function $f$ has a local maximum or a local minimum.
0500-8. Let \( f(x) = x^8 e^{x^2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), what does the Second Derivative Test tell you about that critical number?

c. For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
0500-9. Sketch the graph of a function

\[ H : [0, 8] \rightarrow \mathbb{R} \]

with the following properties:

- \( H \) is continuous on \([0, 8]\);
- \( H'' \) is continuous on \((0, 8)\);
- \( H(0) = H(4) = H(8) = 0 \);
- \( H'(2) = H'(6) = 0 \);
- \( H'' < 0 \) on \((0, 4)\);
- \( H'' > 0 \) on \((4, 8)\).

0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \)

s.t. \( g \) attains a local max value of 20 at \(-3\)
and a local min value of \(-16\) at 3.
0500-11. Let \( f(x) = 2 + \sin^2 x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-12. Let \( f(x) = \ln(x^2 + 1) \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).
c. Find all max intervals of incr/decr for \( f \).
d. Find all max intervals of cc up/cc dn for \( f \).
e. Sketch the graph of \( f \).
Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = x^4 + 2x^3 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).
c. Find all max intervals of incr/decr for \( f \).
d. Find all max intervals of cc up/cc dn for \( f \).
e. Sketch the graph of \( f \).
0500-16. Let \( f(x) = \sqrt{x^2 + 6x + 5}. \)

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). 

Also:

   (i) What is the domain of \( f \)?  
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).   
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-17. Let \( f(x) = x - \sin x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = 2xe^{-x^2/2} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-19. Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).