CALCULUS
Linear approximation
OLD
0540-1. Find the linearization of

\[ f(x) = 2x^3 - 5x \]

at \( x = 2 \).

That is, find \( m \) and \( a \) s.t. the linear function

\[ L(x) = a + m(x - 2) \]

has the same 1-jet at \( x = 2 \) as does \( f(x) \).

That is, find \( m \) and \( a \) s.t. the linear function

\[ L(x) = a + m(x - 2) \]

satisfies: \( L(2) = f(2) \) and \( L'(2) = f'(2) \).
0540-2. Find the linearization of

\[ f(x) = \sec x \]

at \( x = \pi/4 \).

That is, find \( m \) and \( a \) s.t. the linear function

\[ L(x) = a + m(x - (\pi/4)) \]

has the same 1-jet at \( x = \pi/4 \) as does \( f(x) \).

That is, find \( m \) and \( a \) s.t. the linear function

\[ L(x) = a + m(x - (\pi/4)) \]

satisfies:

\[ L(\pi/4) = f(\pi/4) \]

and \( L'(\pi/4) = f'(\pi/4) \).
0540-3. Let \( y = \frac{x^2 \cos x}{e^x} \).

Compute \( \Delta y \) and \( dy \).

0540-4. Let \( u = \frac{w + 4}{\cos(2w + 8)} \).

Compute \( \Delta u \) and \( du \).

0540-5. Let \( z = \frac{e^{2v^2}}{\tan(4v - 1)} \).

a. Compute \( [\Delta z]_{v: \to 0}, \Delta v: \to 0.001 \).

b. Compute \( [dz]_{v: \to 0}, dv: \to 0.001 \).
0540-6. a. Compute \((3.001)^8\).

b. Approx. \((3.001)^8\) by differentials.

c. Let \(L(x)\) be the linearization of \(f(x) = x^8\) at \(x = 3\). Compute \(L(3.001)\).

0540-7. Let \(\theta\) be the number of radians in \(29.9^\circ\). Approximate \(\cos \theta\) by differentials.

0540-8. Approx. \(e^{0.05}\) by differentials.
0540-9. We need to paint a cube whose side length is 10 meters. The coat of paint is to be 0.001 meters thick, so, after painting, the sides will have length 10.002 meters.

a. Let $V = s^3$. Compute $\triangle V$ and $dV$.

b. Using $\triangle V$, compute the exact volume of paint that will be needed.

c. Using $dV$, estimate the volume of paint that will be needed.

d. Compute 0.001 times the surface area of a cube of side length 10 meters.
0540-10. A regular tetrahedron of height $h$ has volume $\frac{\sqrt{3}}{8}h^3$.

Pharaoh asks us to build a pyramid in the shape of a regular tetrahedron, whose height is $300 \pm 1$ feet.

Up to some error, its volume will be $\frac{\sqrt{3}}{8}(300)^3$ cubic feet.

Using differentials, estimate that error.