CALCULUS
Definite integration and Riemann sum problems
OLD
0590-1. Let \( f(x) = 4 - x^2 \).

a. Compute \( L_4 S^2_{-2} f \).
   Sketch \( f \) over \([-2, 2]\) and add, into your sketch, the four rectangles represented by \( L_4 S^2_{-2} f \).

b. Compute \( M_4 S^2_{-2} f \).
   Sketch \( f \) over \([-2, 2]\) and add, into your sketch, the four rectangles represented by \( M_4 S^2_{-2} f \).

c. Compute \( R_4 S^2_{-2} f \).
   Sketch \( f \) over \([-2, 2]\) and add, into your sketch, the four rectangles represented by \( R_4 S^2_{-2} f \).
0590-2. Let \( f(x) = e^x + 5 \).

a. Compute \( L_3 S_0^6 f \) to three decimal places.
b. Compute \( M_3 S_0^6 f \) to three decimal places.
c. Compute \( R_3 S_0^6 f \) to three decimal places.

0590-3. Let \( f(x) = \sin^2 x \).

a. Compute \( L_5 S_0^\pi f \) to three decimal places.
b. Compute \( M_5 S_0^\pi f \) to three decimal places.
c. Compute \( R_5 S_0^\pi f \) to three decimal places.
A car’s acceleration is positive from time 0 to time 12 seconds, and its velocity at various times is given in the table below.

<table>
<thead>
<tr>
<th>time (secs)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>20</td>
<td>35</td>
<td>45</td>
<td>53</td>
<td>59</td>
<td>62</td>
</tr>
</tbody>
</table>

Find upper and lower estimates for the distance traveled by the car over these 12 seconds.
The graph of a function $f$ appears below.

Estimate $\int_{0}^{10} f(x) \, dx$ by computing

(a) $L_5 S_{0}^{10} f$,  
(b) $M_5 S_{0}^{10} f$

and

(c) $R_5 S_{0}^{10} f$.  

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0590-6. Express the area under \( y = e^{-x^2} \) from \( x = -1 \) to \( x = 1 \) as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-7. Express the area under \( y = \sqrt{x^3 + x + 5} \) from \( x = 2 \) to \( x = 4 \) as a limit of left endpoint Riemann sums. (Don’t evaluate the limit.)

0590-8. Express the area under \( y = \cos(x^3) \) from \( x = -2 \) to \( x = 5 \) as a limit of right endpoint Riemann sums. (Don’t evaluate the limit.)
0590-9. Express \( \int_{5}^{13} \frac{e^{-x^2/2}}{\sqrt{2\pi}} \, dx \) as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

0590-10. Let \( f(x) = x^3 \).

a. Write \( R_n S_0^2 f \) as a rational expression in \( n \) (i.e., as one polynomial in \( n \) divided by another).

b. Compute \( \lim_{n \to \infty} R_n S_0^2 f \).
The limit

\[
\lim_{n \to \infty} \left[ \frac{4}{n} \sum_{j=1}^{n} (e^{\sin(3+(4/n)j)}) \right]
\]

represents the area under \( y = f(x) \) from \( x = a \) to \( x = b \), for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.
0590-12. The limit
\[
\lim_{n \to \infty} \left[ \frac{7}{n} \sum_{j=0}^{n-1} \cos \left( e^{2 + \left( \frac{7}{n} \right) j} \right) \right]
\]
represents the area under \( y = f(x) \)
from \( x = a \) to \( x = b \),
for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.
Let \( f(x) = 3 + \sqrt{4 - x^2} \).

a. Sketch the graph of \( y = f(x) \).

b. Compute \( \int_{-2}^{2} f(x) \, dx \), by interpreting this integral as an area.