CALCULUS
Volume by slices and the disk and washer methods:
Problems
OLD
0720-1. Let \( R \) be the region bounded by \( y = x + 1 \) and \( x = 2 \) in \( 1 \leq y \leq 2 \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

0720-2. Let \( R \) be the region bounded by \( y - 1 = (x - 1)^2 \) and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.
0720-3. Let \( R \) be the region bounded by
\[ y = \ln x, \quad x = 4 \text{ and } y = 1. \]

(a) Sketch \( R \).

(b) Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

0720-4. Let \( R \) be the region bounded by
\[ y = \sin x \text{ and } y = 0 \text{ in } 0 \leq x \leq \pi. \]

(a) Sketch \( R \).

(b) Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

Hint: \[ \sin^2 x = \frac{1 - \left[ \cos(2x) \right]}{2} \]
0720-5. Let $R$ be the region bounded by
$$x^2 + (y - 3)^2 = 1.$$ 

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that $2 \int_{-1}^{1} \sqrt{1 - x^2} \, dx$ is known; it is the area enclosed in a circle of radius 1.
0720-6. Let $R$ be the region bounded by $y = x^3$ and $x = y^4$.

a. Sketch $R$.
b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.
c. Find the volume of the solid obtained by rotating $R$ about the line $x = -1/3$.

0720-7. Let $R$ be the region bounded by $y = x^2$ and $x = y^6$.

a. Sketch $R$.
b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.
c. Find the volume of the solid obtained by rotating $R$ about the line $x = -1/3$. 

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Let $R$ be the region bounded by $y = 4 \cos x$, $y = e^x$ in $0 \leq x \leq \pi/4$. Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating $R$ about the line $y = 5$.

Describe the solid of revolution whose volume is given by

$$\pi \int_1^2 \left(9e^{8x} - 4e^{2x}\right) \, dx.$$ Do not evaluate this integral.

Describe the solid of revolution whose volume is given by

$$\pi \int_{\pi/2}^\pi (2 + \sin x)^2 - 4 \, dx.$$ Do not evaluate this integral.
0720-11. A solid $S$ sits above a horizontal plane $P$. $\forall x \geq 0$, let $P_x$ be the horizontal plane that is $x$ units above $P$. Suppose that $S$ lies between $P_1$ and $P_2$. Suppose, also, that $\forall x \in [1, 2]$, the intersection of $S$ and $P_x$ is the region inside an ellipse whose major axis has length $x$ and whose minor axis has length $e^{2x^2}$.

Compute the volume of $S$.

Hint: Remember that if $a$ and $b$ are the major and minor axes of an ellipse $E$, then the area inside $E$ is $\pi ab/4$. 
Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.
We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.