CALCULUS
The derivative of a function is a function
OLD
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

**ANSWER:**

Choose red, **green** or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

Choose red, green or purple.
The graph of $f$ is shown above. Which of the following is the graph of $f'$?

**ANSWER:**

Choose red, green or purple. 

The graph of $f$ is shown above. Freehand a sketch of the graph of $f'$. On your graph, indicate 1 and $-1$ on the horizontal axis.
OLD

ANSWER:
The graph of $f$ is shown above.

a. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not defined?

b. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not continuous?

c. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not differentiable?
The graph of $f$ is shown above.

a. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not defined?  
ANS: $-3, 2$

b. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not continuous?  
ANS: $-3, 1, 2$

c. At which of the numbers $-3, -2, -1, 0, 1, 2, 3$ is $f$ not differentiable?  
ANS: $-3, -2, 1, 2$
The graphs of \( f, f' \) and \( f'' \) are shown above. Which is which?

State the color of \( f \), the color of \( f' \) and the color of \( f'' \).
The graphs of $f$, $f'$ and $f''$ are shown above. Which is which?

State the color of $f$, the color of $f'$ and the color of $f''$. 

ANSWER:

$f''$ is red

$f'$ is green

$f$ is blue
Let \( f(s) = 7s - 5s^3 \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, and using the cubic binomial formula
\[
(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,
\]
compute \( f'(s) \).

c. What is the domain of the derivative \( f' \)?
0280-6. Let $f(s) = 7s - 5s^3$.

a. What is the domain of $f$?

**ANSWER:**

a. $\mathbb{R} = (-\infty, \infty)$
0280-6. Let \( f(s) = 7s - 5s^3 \).

b. Using the definition of the derivative, and using the cubic binomial formula

\[(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,\]

compute \( f'(s) \).

**ANSWER:**

\[
b. \quad f'(s) = \lim_{h \to 0} \frac{[7(s + h) - 5(s + h)^3] - [7s - 5s^3]}{h}
\]

\[
= \lim_{h \to 0} \frac{[7(s + h) - 5(s^3 + 3s^2h + 3sh^2 + h^3)] - [7s - 5s^3]}{h}
\]

\[
= \lim_{h \to 0} \frac{7h - 5(3s^2h + 3sh^2 + h^3)}{h}
\]

\[
= \lim_{h \to 0} 7 - 5(3s^2 + 3sh + h^2)
\]

\[= 7 - 5(3s^2) = 7 - 15s^2\]
Let \( f(s) = 7s - 5s^3 \).

c. What is the domain of the derivative \( f' \)?

**ANSWER:**

\[ f'(s) = 7 - 15s^2 \]

c. \( \mathbb{R} = (-\infty, \infty) \)
0280-7. Let \( f(x) = \frac{1 + x}{2 + x} \).

a. What is the domain of \( f \)?

b. Using the definition of the derivative, compute \( f'(x) \).

c. What is the domain of the derivative \( f' \)?
Let \( f(x) = \frac{1 + x}{2 + x} \).

a. What is the domain of \( f \)?

**ANSWER:**

a. \( \mathbb{R} \setminus \{-2\} \)
Let \( f(x) = \frac{1 + x}{2 + x} \).

b. Using the definition of the derivative, compute \( f'(x) \).

**ANSWER:**

\[
\begin{align*}
1 & \left[ \frac{1 + x + h}{2 + x + h} - \frac{1 + x}{2 + x} \right] \\
= & \frac{1}{h} \left[ \frac{(1 + x + h)(2 + x) - (2 + x + h)(1 + x)}{(2 + x + h)(2 + x)} \right] \\
= & \frac{h(2 + x) - h(1 + x)}{h(2 + x + h)(2 + x)} \\
= & \frac{1}{h(2 + x + h)(2 + x)} \quad h \rightarrow 0 \\
= & \frac{1}{(2 + x)^2}
\end{align*}
\]
0280-7. Let $f(x) = \frac{1 + x}{2 + x}$.

c. What is the domain of the derivative $f'$?

**ANSWER:** $f'(x) = \frac{1}{(2 + x)^2}$

c. $\mathbb{R}\setminus\{-2\}$
Let \( f(x) = |x^2 - 2x - 3| \).

At which numbers is \( f \) not differentiable?

**Hint:** Determine the (maximal) intervals where \( x^2 - 2x - 3 \) is positive and negative.

Sketch the graph of \( y = x^2 - 2x - 3 \).

Sketch the graph of \( y = f(x) \).

**GENERAL RULE:**

At numbers \( x \) where \( x^2 - 2x - 3 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.
Let \( f(x) = |x^2 - 2x - 3| \).

At which numbers is \( f \) not differentiable?

**ANSWER:**

\[
x^2 - 2x - 3 = (x + 1)(x - 3)
\]

- positive on \( 3 < x \)
- negative on \( -1 < x < 3 \)
- positive on \( x < -1 \)
Let \( f(x) = |x^2 - 2x - 3| \).

At which numbers is \( f \) not differentiable?

**ANSWER:**

\[ x^2 - 2x - 3 = (x + 1)(x - 3) \]

- positive on \( 3 < x \)
- negative on \( -1 < x < 3 \)
- positive on \( x < -1 \)

\( f \) is not differentiable at \(-1\) and \(3\).
Let \( f(x) = |x^4 - 2x^3 - 3x^2| \).

At which numbers is \( f \) not differentiable?

**Hint:** Determine the (maximal) intervals where \( x^4 - 2x^3 - 3x^2 \) is positive and negative.

Sketch the graph of \( y = x^4 - 2x^3 - 3x^2 \).

Sketch the graph of \( y = f(x) \).

**GENERAL RULE:**

At numbers \( x \) where \( x^4 - 2x^3 - 3x^2 \) has a root of multiplicity one, \( f \) is not differentiable. Everywhere else, \( f \) is differentiable.
0280-9. Let \( f(x) = |x^4 - 2x^3 - 3x^2| \).

At which numbers is \( f \) not differentiable?

**ANSWER:**

\[
x^4 - 2x^3 - 3x^2 = (x + 1)x^2(x - 3)
\]

- positive on \( 3 < x \)
- negative on \( 0 < x < 3 \)
- negative on \( -1 < x < 0 \)
- positive on \( x < -1 \)
Let \( f(x) = |x^4 - 2x^3 - 3x^2| \). At which numbers is \( f \) not differentiable?

**Answer:**

\[
x^4 - 2x^3 - 3x^2 = (x + 1)x^2(x - 3)
\]

- positive on \( 3 < x \)
- negative on \( 0 < x < 3 \)
- negative on \( -1 < x < 0 \)
- positive on \( x < -1 \)

\[
y = f(x) = |x^4 - 2x^3 - 3x^2|
\]

\( f \) is not differentiable at \(-1\) and \(3\).