CALCULUS
Derivatives of inverse functions
(The Inverse Function Theorem)
OLD
0440-1. Differentiate \( y = \arctan \left( e^x + \sqrt[3]{x} \right) \).

**ANSWER:** \( y = \arctan \left( e^x + x^{1/3} \right) \)

\[
\frac{dy}{dx} = \frac{e^x + (1/3)x^{-2/3}}{1 + \left( e^x + x^{1/3} \right)^2}
\]

0440-2. Differentiate \( F(t) = \left[ e^{3t+4} \right] \left[ \arcsin \left( t^2 \right) \right] \).

**ANSWER:**

\[
F'(t) = \left[ (e^{3t+4}) \cdot (3) \right] \left[ \arcsin \left( t^2 \right) \right] + \left[ e^{3t+4} \right] \left[ \frac{2t}{1 + (t^2)^2} \right]
\]
Differentiate $f(x) = \sin(\arctan x)$.

\[ f(x) = \frac{x}{\sqrt{1 + x^2}} \]

\[ f'(x) = \frac{\left(1 + x^2\right)^{1/2}}{1 + x^2} \left[ 1 - x \left( \frac{1}{2} \right) (1 + x^2)^{-1/2} (2x) \right] \]

\[ = \frac{1 + x^2}{(1 + x^2)^{3/2}} - \frac{x \left[ (1/2) (2x) \right]}{(1 + x^2)^{3/2}} = \frac{1}{(1 + x^2)^{3/2}} \]
Differentiate \( f(x) = \sin(\arctan x) \).

SECOND ANSWER:

\[
f'(x) = \left[ \cos(\arctan x) \right] \left[ \frac{1}{1 + x^2} \right]
\]

\[
\tan \theta = x
\]

\[
\arctan x = \theta
\]

\[
\cos(\arctan x) = \cos \theta = \frac{1}{\sqrt{1 + x^2}}
\]

\[
f'(x) = \left[ \frac{1}{(1 + x^2)^{1/2}} \right] \left[ \frac{1}{1 + x^2} \right] = \frac{1}{(1 + x^2)^{3/2}}
\]

\[
f'(x) = \frac{1}{(1 + x^2)^{3/2}}
\]
Differentiate \( v(s) = \arccot \left( \frac{\sqrt{2-s}}{2+s} \right) \).

**Answer:**

\[
v'(s) = -\frac{1}{2} \left( \frac{2-s}{2+s} \right)^{-1/2} \left[ \frac{(2+s)(-1)-(2-s)(1)}{(2+s)^2} \right]
\]

\[
= -\frac{1}{2} \left( \frac{2+s}{2-s} \right)^{1/2} \left[ \frac{-2s-2+s}{(2+s)^2} \right]
\]

\[
= -\frac{1}{2} \left( \frac{2+s}{2-s} \right)^{1/2} \left[ \frac{-4}{(2+s)^2} \right]
\]
Differentiate \( v(s) = \arccot \left( \sqrt{\frac{2-s}{2+s}} \right) \).

\[
v'(s) = -\frac{\frac{1}{2} \left( \frac{2+s}{2-s} \right)^{1/2}}{2+s} \cdot \frac{-4}{(2+s)^2}
\]

\[
= \frac{1}{2} \left( \frac{2+s}{2-s} \right)^{1/2} \cdot \frac{1}{(2+s)^2} \cdot \frac{1}{2+s}
\]

\[
= \frac{1}{2} \left[ \frac{2+s}{2-s} \right]^{1/2} \cdot \left[ \frac{1}{(2+s)^2} \right]^{1/2}
\]
Differentiate $v(s) = \arccot \left[ \sqrt{\frac{2 - s}{2 + s}} \right]$.

**ANSWER:**

$$v'(s) = \frac{1}{2} \left[ \frac{2+s}{2-s} \right]^{1/2} \left[ \frac{1}{(2+s)^2} \right]^{1/2}$$

$$= \frac{1}{2} \left[ \frac{2+s}{(2-s)(2+s)^2} \right]^{1/2}$$

$$= \frac{1}{2} \left[ \frac{1}{(2-s)(2+s)} \right]^{1/2}$$

$$= \frac{1}{2} \left[ \frac{1}{4-s^2} \right]^{1/2}$$

$$= \frac{1}{2 \sqrt{4-s^2}}$$
Draw a graph of a 1-1 function \( f \)
which passes through \((4, 5)\)
and whose tangent line at \((4, 5)\) has slope \(2/3\).
In the same picture,
**draw** that tangent line.
In the same picture,
**draw** a right triangle whose hypotenuse is on the tangent line and whose legs have lengths 2 and 3.
In a separate picture, **reflect**
through the \(45^\circ\) line,
everything in the previous picture.

Let \( g := f^{-1} \).

What are the values of \( f(4) \) and \( f'(4) \)?

What are the values of \( g(5) \) and \( g'(5) \)?
0440-5. Draw a graph of a 1-1 function $f$ which passes through $(4, 5)$ and whose tangent line at $(4, 5)$ has slope $2/3$. In the same picture, draw that tangent line.

**ANSWER:**
OLD

0440-5. In the same picture, draw a right triangle whose hypotenuse is on the tangent line and whose legs have lengths 2 and 3.

ANSWER:
0440-5. In a separate picture, reflect, through the 45° line, everything in the previous picture.

**ANSWER:**
Let $g := f^{-1}$.

What are the values of $f(4)$ and $f'(4)$?

What are the values of $g(5)$ and $g'(5)$?

**Answer:**

$f(4) = 5$, $f'(4) = \frac{2}{3}$

g(5) = 4, $g'(5) = \frac{3}{2}$