CALCULUS
Even more graphing problems
OLD
0500-1. Let $f : [0, 15) \setminus \{8\} \rightarrow \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   (ii) $f$ is decreasing;
   (iii) $f$ is concave up;
   and (iv) $f$ is concave down.

b. Find all points of inflection for $f$. 

0500-1. Let $f : [0, 15) \setminus \{8\} \to \mathbb{R}$ be as shown.

a. Find the maximal intervals on which
   
   (i) $f$ is increasing;
   
   (ii) $f$ is decreasing;

**ANSWER:**

a. (i) incr on $[0,2]$, on $[6,8)$ and on $[11,13]$

a. (ii) decr on $[2,6]$, on $(8,11]$ and on $[13,15]$
0500-1. Let \( f : [0, 15) \backslash \{8\} \rightarrow \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (iii) \( f \) is concave up;
   and (iv) \( f \) is concave down.

b. Find all points of inflection for \( f \).

ANS:  
   a. (iii) cc up on \([4,8)\) and on \((8,12]\)
   a. (iv) cc dn on \([0,4]\) and on \([12,15)\)
   b. points of inflection: \((4,3)\) and \((12,4)\)
0500-2. Let $f : [0, 15) \setminus \{8\} \to \mathbb{R}$ be continuous from the right at 0. The graph of $f'$ is shown below. Find the maximal intervals on which
(i) $f$ is concave up;
and (ii) $f$ is concave down.

**ANSWER:**

(i) $f$ is cc up on $[0, 2]$, on $[6, 8)$ and on $[11, 13]$

(ii) $f$ is cc dn on $[2, 6]$, on $(8, 11]$ and on $[13, 15]$
0500-3. Let \( f : [0, 14] \to \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.
0500-3. Let \( f : [0, 14] \rightarrow \mathbb{R} \) be as shown.

a. Find the maximal intervals on which
(i) \( f \) is increasing;
and (ii) \( f \) is decreasing.

**ANSWER:** a.

(i) \( f \) is incr on \([0, 3]\) and on \([10, 14]\)
(ii) \( f \) is decr on \([3, 10]\)
0500-3. Let \( f : [0, 14] \to \mathbb{R} \) be as shown.

b. Find all numbers at which
(i) \( f \) attains a local maximum;
and (ii) \( f \) attains a local minimum.

**ANSWER:** b.
(i) \( f \) attains a local maximum at 3.
(ii) \( f \) attains a local minimum at 10.
Let $f$ be continuous on $[0, 14]$.

The graph of $f'$ is shown below.

a. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.

b. At what numbers does $f$ have
   (i) a local maximum?
   (ii) a local minimum?
0500-4. Let \( f \) be continuous on \([0, 14]\).

The graph of \( f' \) is shown below.

a. Find the maximal intervals on which
   (i) \( f \) is concave up;
   and (ii) \( f \) is concave down.

**ANSWER:** a.
   (i) \( f \) is cc up on \([0, 3]\) and on \([10, 14]\).
   (ii) \( f \) is cc dn on \([3, 10]\).
0500-4. Let $f$ be continuous on $[0, 14]$. The graph of $f'$ is shown below.

b. At what numbers does $f$ have
   (i) a local maximum? (ii) a local minimum?

**ANSWER:** b.
   (i) $f$ has a local maximum at 5.
   (ii) $f$ has a local minimum at 1 and at 13.
Let $f(x) = x^4 - 4x^3 + 4x^2 + 9$.

a. Find the maximal intervals on which
   (i) $f$ is increasing;
   and (ii) $f$ is decreasing.

b. Find all numbers at which
   (i) $f$ attains a local maximum;
   and (ii) $f$ attains a local minimum.

c. Find the maximal intervals on which
   (i) $f$ is concave up;
   and (ii) $f$ is concave down.
0500-5. Let \( f(x) = x^4 - 4x^3 + 4x^2 + 9 \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

**ANSWER:** \( f'(x) = 4x^3 - 12x^2 + 8x \)

\[ = 4x(x - 1)(x - 2) \]

a. (i) \( f \) is increasing on \([0, 1]\) and on \([2, \infty)\).
   (ii) \( f \) is decreasing on \((-\infty, 0]\) and on \([1, 2]\).

b. (i) \( f \) attains a local maximum at 1.
   (ii) \( f \) attains a local minimum at 0 and at 2.
Let \( f(x) = x^4 - 4x^3 + 4x^2 + 9 \).

c. Find the maximal intervals on which

(i) \( f \) is concave up; and

(ii) \( f \) is concave down.

**ANSWER:**
\[
\begin{align*}
  f'(x) &= 4x^3 - 12x^2 + 8x \\
  f''(x) &= 12x^2 - 24x + 8 \\
        &= 4(3x^2 - 6x + 2)
\end{align*}
\]

\[
\begin{align*}
  r &:= \frac{6 - \sqrt{36 - 24}}{6}, & s &:= \frac{6 + \sqrt{36 - 24}}{6}
\end{align*}
\]

roots of \( f'' \): \( r \) and \( s \)

\[c. \ (i) \ f \text{ is concave up on } (-\infty, r] \text{ and on } [s, \infty). \]

\[c. \ (ii) \ f \text{ is concave down on } [r, s].\]
0500-6. Let $f(x) = (x^2 + 1)e^{-x}$.

a. Find the maximal intervals on which
   (i) $f$ is increasing; and
   (ii) $f$ is decreasing.

b. Find all numbers at which
   (i) $f$ attains a local maximum; and
   (ii) $f$ attains a local minimum.

c. Find the maximal intervals on which
   (i) $f$ is concave up; and
   (ii) $f$ is concave down.

d. Find all points of inflection for $f$. 
0500-6. Let \( f(x) = (x^2 + 1)e^{-x} \).

a. Find the maximal intervals on which
   (i) \( f \) is increasing;
   and (ii) \( f \) is decreasing.

b. Find all numbers at which
   (i) \( f \) attains a local maximum;
   and (ii) \( f \) attains a local minimum.

**ANSWER:**

\[
f'(x) = (2x)e^{-x} + (x^2 + 1)e^{-x}(-1)
= -(x^2 - 2x + 1)e^{-x}
= -(x - 1)^2e^{-x}
\]

a. (i) \( f \) has no intervals of increase.
   (ii) \( f \) is decreasing on \( \mathbb{R} = (-\infty, \infty) \).

b. (i) \( f \) has no local maxima.
   (ii) \( f \) has no local minima.
0500-6. Let \( f(x) = (x^2 + 1)e^{-x} \).

**c. Find** the maximal intervals on which

(i) \( f \) is concave up;

and (ii) \( f \) is concave down.

**d. Find all points of inflection for** \( f \).

**ANSWER:**

\[
\begin{align*}
  f'(x) &= -(x^2 - 2x + 1)e^{-x} \\
  f''(x) &= -(2x - 2)e^{-x} - (x^2 - 2x + 1)e^{-x}(-1) \\
         &= (x^2 - 4x + 3)e^{-x} \\
         &= (x - 1)(x - 3)e^{-x}
\end{align*}
\]

**c. (i) \( f \) is cc up on \((-\infty, 1]\) and on \([3, \infty)\).**

(ii) \( f \) is cc down on \([1, 3]\).

**d.** Points of inflection for \( f \):

\( (1, 2/e), \quad (3, 10/e^3) \)
0500-7. Let \( f(x) = xe^{-x^2/2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), use the Second Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.

**ANSWER:**

\[
f'(x) = e^{-x^2/2} + xe^{-x^2/2}(-2x/2) = (1 - x^2)e^{-x^2/2}
\]

\[
f''(x) = (-2x)e^{-x^2/2} + (1 - x^2)e^{-x^2/2}(-2x/2) = (x^3 - 3x)e^{-x^2/2}
\]
Let $f(x) = xe^{-x^2/2}$.

a. Find all critical numbers for $f$.

b. For each critical number for $f$, use the Second Derivative Test to determine whether, at that number, the function $f$ has a local maximum or a local minimum.

**ANSWER:**

$$f'(x) = (1 - x^2)e^{-x^2/2}$$

$$f''(x) = (x^3 - 3x)e^{-x^2/2}$$

a. Critical numbers for $f$: $-1$ and $1$

b. $f''(-1) = (-1 + 3)e^{-1/2} > 0$, so $f$ has a local minimum at $-1$.

$$f''(1) = (1 - 3)e^{-1/2} < 0$$

so $f$ has a local maximum at $1$. ■
0500-8. Let \( f(x) = x^8 e^{x^2} \).

a. Find all critical numbers for \( f \).

b. For each critical number for \( f \), what does the Second Derivative Test tell you about that critical number?

c. For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.
Let $f(x) = x^8 e^{x^2}$.

a. Find all critical numbers for $f$.

**ANSWER:**

\[
f'(x) = (8x^7)e^{x^2} + x^8 e^{x^2}(2x)
\]

\[
= (2x^9 + 8x^7)e^{x^2}
\]

\[
= x^7(2x^2 + 8)e^{x^2}
\]

a. critical numbers: 0
0500-8. Let \( f(x) = x^8 e^{x^2} \).

b. For each critical number for \( f \), what does the Second Derivative Test tell you about that critical number?

**ANSWER:** critical numbers: 0

\[
f'(x) = (2x^9 + 8x^7)e^{x^2}
\]

\[
f''(x) = (18x^8 + 56x^6)e^{x^2} + (2x^9 + 8x^7)e^{x^2}(2x)
\]

b. \( f''(0) = 0 \)

The second derivative test gives **NO** information.
0500-8. Let \( f(x) = x^8 e^{x^2} \).

c. For each critical number for \( f \), use the First Derivative Test to determine whether, at that number, the function \( f \) has a local maximum or a local minimum.

**ANSWER:** critical numbers: 0

\[
f'(x) = (2x^9 + 8x^7)e^{x^2}
= x^7(2x^2 + 8)e^{x^2}
\]

c. \( f \) is decreasing on \((-\infty, 0]\).
\( f \) is increasing on \([0, \infty)\).

Then \( f \) has a local minimum at 0.
0500-9. Sketch the graph of a function

\[ H : [0, 8] \to \mathbb{R} \]

with the following properties:

1. \( H \) is continuous on \([0, 8]\);
2. \( H'' \) is continuous on \((0, 8)\);
3. \( H(0) = H(4) = H(8) = 0 \);
4. \( H'(2) = H'(6) = 0 \);
5. \( H'' < 0 \) on \((0, 4)\);
6. \( H'' > 0 \) on \((4, 8)\).

**ANSWER:**

There are many other answers.
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \)

s.t. \( g \) attains a local max value of 20 at \(-3\) and a local min value of \(-16\) at 3.

**ANSWER:**

\[
g'(t) = 3at^2 + 2bt + c
\]

\[
0 = g'(-3) = 27a - 6b + c
\]

\[
0 = g'(3) = 27a + 6b + c
\]

\[
0 = [g'(3)] - [g'(-3)] = 12b
\]

\[
0 = b
\]

\[
0 = g'(3) = 27a + (6)(0) + c
\]

\[-27a = c\]
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \) s.t. \( g \) attains a local max value of 20 at \(-3\) and a local min value of \(-16\) at 3.

**ANSWER:** \( 0 = b \quad -27a = c \)

\[
g(t) = at^3 + bt^2 + ct + d = at^3 - 27at + d
\]

\[
20 = g(-3) = -27a + 81a + d = 54a + d
\]

\[
-16 = g(3) = 27a - 81a + d = -54a + d
\]

\[
4 = [g(3)] + [g(-3)] = 2d
\]

\[
2 = d
\]

\[
20 = g(-3) = 54a + d = 54a + 2
\]

\[
18 = 54a
\]

\[
1/3 = 18/54 = a
\]
0500-10. Find a cubic \( g(t) = at^3 + bt^2 + ct + d \) s.t. \( g \) attains a local max value of 20 at -3 and a local min value of -16 at 3.

**Answer:** \( 0 = b \quad -27a = c \quad 2 = d \)

\[
\frac{1}{3} = a \\
-9 = -27(\frac{1}{3}) = -27a = c
\]

\[
g(t) = at^3 + bt^2 + ct + d = (\frac{1}{3})t^3 - 9t + 2
\]
0500-11. Let \( f(x) = 2 + \sin^2 x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
Let \( f(x) = 2 + \sin^2 x \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

a. \( f \) is even and \( \pi \)-periodic.

b. (i) \( \text{dom}[f] = \mathbb{R} \supseteq [0, \pi/2) \)

(ii) \( f(0) = 2 \) is the \( y \)-intercept.

\( \forall x \in \mathbb{R}, f(x) \neq 0 \), so no \( x \)-intercepts

\( f \) is positive on \( \mathbb{R} \).

(iii) no vertical or horizontal asymptotes
0500-11. Let \( f(x) = 2 + \sin^2 x \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:** \( f \) is even and \( \pi \)-periodic.

c. \( f'(x) = 2(\sin x)(\cos x) \)

is positive on \( 0 < x < \pi/2 \).

\( f \) is increasing on \([0, \pi/2]\).

\( f \) even gives: \( f \) decreasing on \([-\pi/2, 0]\)

\( f \) \( \pi \)-periodic gives: \( f \) incr on \([n\pi, n\pi + (\pi/2)]\), \( f \) decr on \([n\pi - (\pi/2), n\pi]\), \( \forall n \in \mathbb{Z} \).
0500-11. Let \( f(x) = 2 + \sin^2 x \).

\( \text{d. Find all max intervals of cc up/cc dn for } f. \)

**ANSWER**: \( f \) is even and \( \pi \)-periodic.

\[ f'(x) = 2(\sin x)(\cos x) \]

\[ f''(x) = 2(\cos x)(\cos x) + 2(\sin x)(-\sin x) \]

\[ = 2[(\cos^2 x) - (\sin^2 x)] \]

\[ = 2 \cos(2x) \]

is positive on \(-\pi/4 < x < \pi/4\)

and negative on \(\pi/4 < x < 3\pi/4\).

\( f \) is concave up on \([-\pi/4, \pi/4]\)

and concave down on \([\pi/4, 3\pi/4]\).
Let $f(x) = 2 + \sin^2 x$.

d. Find all max intervals of cc up/cc dn for $f$.

**ANSWER:** $f$ is even and $\pi$-periodic.

d. $f$ is concave up on $[-\pi/4, \pi/4]$ and concave down on $[\pi/4, 3\pi/4]$.

$\pi$-periodicity gives:

$f$ concave up on $[n\pi - (\pi/4), n\pi + (\pi/4)]$, $f$ concave down on $[n\pi + (\pi/4), n\pi + (3\pi/4)]$, $\forall n \in \mathbb{Z}$
0500-11. Let $f(x) = 2 + \sin^2 x$.

e. Sketch the graph of $f$.

ANSWER:

e.
0500-12. Let \( f(x) = \ln(x^2 + 1) \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:
(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-12. Let \( f(x) = \ln(x^2 + 1). \)

a. Describe the symmetries, if any, of \( f. \)

b. Find all max intervals of pos/neg for \( f. \)

Also:

(i) What is the domain of \( f? \)

(ii) Find all \( x- \) and \( y- \)intercepts of \( f. \)

(iii) Find all vert/horiz asymptotes of \( f. \)

**ANSWER:**
a. \( f \) is even.

b. (i) \( \text{dom}[f] = \mathbb{R} \)

   (ii) \( f(0) = 0 \) is the \( y \)-intercept.

   \[ x^2 + 1 = 1 \iff x = 0. \]

   \( x \)-intercepts: \( f(x) = 0 \iff x = 0 \)

   \( x^2 + 1 > 1 \) on \( x < 0 \) and on \( 0 < x. \)

   \( f \) is positive on \(( -\infty, 0) \) and on \(( 0, \infty). \)

(iii) no vertical or horizontal asymptotes
0500-12. Let $f(x) = \ln(x^2 + 1)$.

\[ \text{c. Find all max intervals of incr/decr for } f. \]

**ANSWER:** $f$ is even

\[ c. \quad f'(x) = \frac{2x}{x^2 + 1} \] is positive on $0 < x$.

$f$ is increasing on $[0, \infty)$. $f$ even gives: $f$ decreasing on $(-\infty, 0]$.
0500-12. Let $f(x) = \ln(x^2 + 1)$.

d. Find all max intervals of cc up/cc dn for $f$.

ANSWER: $f$ is even

\[ f'(x) = \frac{2x}{x^2 + 1} \]

d. \[ f''(x) = \frac{\left(x^2 + 1\right)(2) - (2x)(2x)}{(x^2 + 1)^2} \] 

\[ = \frac{-2x^2 + 2}{(x^2 + 1)^2} \]
Let \( f(x) = \ln(x^2 + 1) \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is even

\[
d. \quad f''(x) = \frac{-2x^2 + 2}{(x^2 + 1)^2} = \frac{-2(x + 1)(x - 1)}{(x^2 + 1)^2}
\]

is negative on \((-\infty, -1)\)
and positive on \((-1, 1)\)
and negative on \((1, \infty)\).

\( f \) is concave down on \((-\infty, -1]\)
and concave up on \([-1, 1]\)
and concave down on \([1, \infty)\).
0500-12. Let \( f(x) = \ln(x^2 + 1) \).

e. Sketch the graph of \( f \).

ANSWER:

e.
500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

a. Describe the symmetries, if any, of \( f \).
b. Find all max intervals of pos/neg for \( f \).
   Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

**ANSWER:**

a. \( f \) is odd.

b. (i) \( \text{dom}[f] = (-\infty, -1) \cup (1, \infty) \)

   (ii) \( f(0) \) is undefined, so no \( y \)-intercept

   \( \forall x \in \text{dom}[f], f(x) \neq 0 \), so no \( x \)-intercepts

   \( f \) is positive on \((1, \infty)\)

   and negative on \((-\infty, -1)\).
Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

b. Also:
   (iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:** \( f \) is odd.

b. (iii) vertical/horizontal asymptotes:

\[
f(x) = \frac{x}{\sqrt{x^2 - 1}} = \frac{1}{\sqrt{1 - (1/x)^2}} \quad \text{as } x \to 1^+ \to \infty
\]

\( f \) odd gives: \( f(x) \xrightarrow{x \to -1^-} -\infty \)

\( x = -1 \) and \( x = 1 \) are vert. asymptotes.
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

b. Also:
   (iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:** \( f \) is odd.

b. (iii) vertical/horizontal asymptotes:

\[
f(x) = \frac{x}{\sqrt{x^2 - 1}} \quad x \geq 1 \quad \frac{1}{\sqrt{1 - (1/x)^2}} \quad x \to \infty \quad 1
\]

\( f \) odd gives: \( f(x) \xrightarrow{x \to -\infty} -1 \)

\( y = -1 \) and \( y = 1 \) are hor. asymptotes.
Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:** \( f \) is odd.  \( \text{dom}[f] = (-\infty, -1) \cup (1, \infty) \)

c.  \[
f'(x) = \frac{([x^2 - 1]^{1/2})(1) - (x)([1/2][x^2 - 1]^{-1/2}[2x])}{x^2 - 1}
\]

\[
= \frac{[x^2 - 1]^{1/2} - (x^2)[x^2 - 1]^{-1/2}}{x^2 - 1}
\]

\[
= \frac{[x^2 - 1] - (x^2)[1]}{[x^2 - 1]^{3/2}}
\]

\[
= \frac{-1}{[x^2 - 1]^{3/2}}
\]
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

c. Find all max intervals of incr/decr for \( f \).

\textbf{ANSWER:} \( f \) is odd. \quad \text{dom}[f] = (-\infty, -1) \cup (1, \infty)

c. \( f'(x) = \frac{-1}{[x^2 - 1]^{3/2}} \) is negative on \((1, \infty)\).

\( f \) is decreasing on \((1, \infty)\).

\( f \) odd gives: \( f \) is decreasing on \((-\infty, -1)\).
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is odd. \[ \text{dom}[f] = (-\infty, -1) \cup (1, \infty) \]

\[ f'(x) = \frac{-1}{[x^2 - 1]^{3/2}} = -[x^2 - 1]^{-3/2} \]

\[ f''(x) = \left(\frac{3}{2}\right)[x^2 - 1]^{-5/2}[2x] \]

\[ = \frac{3x}{[x^2 - 1]^{5/2}} \]

is negative on \( x < -1 \) and positive on \( x > 1 \).

\( f \) is concave down on \((-\infty, -1)\) and concave up on \((1, \infty)\).
0500-13. Let \( f(x) = \frac{x}{\sqrt{x^2 - 1}} \).

e. Sketch the graph of \( f \).

**ANSWER:**

e.
0500-14. Let $f(x) = x^4 + 2x^3$.

a. Describe the symmetries, if any, of $f$.

b. Find all max intervals of pos/neg for $f$. Also:
   (i) What is the domain of $f$?
   (ii) Find all $x$- and $y$-intercepts of $f$.
   (iii) Find all vert/horiz asymptotes of $f$.

c. Find all max intervals of incr/decr for $f$.

d. Find all max intervals of cc up/cc dn for $f$.

e. Sketch the graph of $f$. 
0500-14. Let \( f(x) = x^4 + 2x^3 \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?
(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

a. **NO** symmetries

b. 

(i) \( \text{dom}[f] = \mathbb{R} \)

(ii) \( f(0) = 0 \) is the \( y \)-intercept.

\( x \)-intercepts: 

\[
[f(x) = 0] \text{ iff } [x = 0 \text{ or } x = -2]
\]

\( f(x) = (x + 2)x^3 \) is positive on \( x < -2 \) and negative on \( -2 < x < 0 \) and positive on \( 0 < x \).

(iii) **vertical/horizontal asymptotes:** none
0500-14. Let \( f(x) = x^4 + 2x^3 \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

c. \( f'(x) = 4x^3 + 6x^2 \)

\( = 4(x + (3/2))x^2 \)

is negative on \( x < -3/2 \)

and positive on \(-3/2 < x < 0\)

and positive on \( 0 < x \).

\( f \) is decreasing on \((-\infty, -3/2] \)

and increasing on \([-3/2, \infty)\).

Also, \( f'(0) = 0 \).
Let $f(x) = x^4 + 2x^3$.

Find all max intervals of cc up/cc dn for $f$.

**Answer:**

$f'(x) = 4x^3 + 6x^2$

$f''(x) = 12x^2 + 12x$

$= 12(x + 1)x$

is positive on $x < -1$
and negative on $-1 < x < 0$
and positive on $0 < x$.

$f$ is concave up on $(-\infty, -1]$ and concave down on $[-1, 0]$ and concave up on $[0, \infty)$. 
0500-14. Let \( f(x) = x^4 + 2x^3 \).

e. Sketch the graph of \( f \).

ANSWER:

e.
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

**ANSWER:**

a. \( f \) is even.

b. \( f(x) = \frac{1}{(x + 2)(x - 2)} \)

(i) \( \text{dom}[f] = \mathbb{R} \setminus \{-2, 2\} \supseteq [0, \infty) \setminus \{2\} \)

(ii) \( f(0) = -1/4 \) is the \( y \)-intercept.

\( \forall x \in \text{dom}[f], f(x) \neq 0 \), so no \( x \)-intercepts

\( f(x) \) is positive on \( x < -2 \)
and negative on \( -2 < x < 2 \)
and positive on \( 2 < x \).
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

(iii) \( \lim_{x \to \infty} f(x) = 0 \) and \( \lim_{x \to -\infty} f(x) = 0 \)

\( y = 0 \) is the only horizontal asymptote.

\( \lim_{x \to -2^-} f(x) = \infty \) and \( \lim_{x \to -2^+} f(x) = -\infty \)

\( f \) even gives:

\( \lim_{x \to 2^-} f(x) = -\infty \) and \( \lim_{x \to 2^+} f(x) = \infty \)

\( x = -2 \) and \( x = 2 \)

are the vertical asymptotes.
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:** \( f \) is even.

c. \( f(x) = \left[ x^2 - 4 \right]^{-1} \)

\[
f'(x) = - \left[ x^2 - 4 \right]^{-2} [2x] = \frac{-2x}{(x + 2)^2 (x - 2)^2}
\]

is negative on \( 0 < x < 2 \) and on \( 2 < x \).

\( f \) is decreasing on \([0, 2)\) and on \((2, \infty)\).

\( f \) even gives:

\( f \) is increasing on \((-\infty, -2)\) and on \((-2, 0]\).
Let \( f(x) = \frac{1}{x^2 - 4} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is even.

\[
f'(x) = \frac{-2x}{(x^2 - 4)^2}
\]

\[
f''(x) = \frac{\left[(x^2 - 4)^2\right] [-2] - [-2x] \left[2 (x^2 - 4)(2x)\right]}{(x^2 - 4)^4}
\]

\[
= \frac{\left[x^2 - 4\right] [-2] - [-2x][4x]}{(x^2 - 4)^3}
\]
Let  \( f(x) = \frac{1}{x^2 - 4} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is even.

d. \( f''(x) = \frac{\left[ x^2 - 4 \right] [-2] - [-2x][4x]}{\left( x^2 - 4 \right)^3} \)

\[
= \frac{\left[ -2x^2 + 8 \right] + \left[ 8x^2 \right]}{\left( x^2 - 4 \right)^3}
\]

\[
= \frac{6x^2 + 8}{\left( x^2 - 4 \right)^3}
\]
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f \) is even.

d. \( f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3} = \frac{6x^2 + 8}{(x + 2)^3 (x - 2)^3} \)

is positive on \( x < -2 \)
and negative on \( -2 < x < 2 \)
and positive on \( 2 < x \).

\( f \) is concave up on \((-\infty, -2)\)
and concave down on \((-2, 2)\)
and concave up on \((2, \infty)\).
0500-15. Let \( f(x) = \frac{1}{x^2 - 4} \).

e. Sketch the graph of \( f \).

**ANSWER:**

e.
0500-16. Let $f(x) = \sqrt{x^2 + 6x + 5}$.

a. Describe the symmetries, if any, of $f$.

b. Find all max intervals of pos/neg for $f$. Also:
   (i) What is the domain of $f$?
   (ii) Find all $x$- and $y$-intercepts of $f$.
   (iii) Find all vert/horiz asymptotes of $f$.

c. Find all max intervals of incr/decr for $f$.

d. Find all max intervals of cc up/cc dn for $f$.

e. Sketch the graph of $f$. 
0500-16. Let $f(x) = \sqrt{x^2 + 6x + 5}$.

a. Describe the symmetries, if any, of $f$.

b. Find all max intervals of pos/neg for $f$.

**ANSWER:**

a. **NO** symmetries

*Note: $f$ is symmetric about $x = -3$, but this is not one of our standard symmetries.*

b. $x^2 + 6x + 5 = (x + 5)(x + 1)$

   is positive on $x < -5$

   and negative on $-5 < x < -1$

   and positive on $-1 < x$. 
0500-16. Let $f(x) = \sqrt{x^2 + 6x + 5}$.

b. Find all max intervals of pos/neg for $f$.

Also:

(i) What is the domain of $f$?

(ii) Find all $x$- and $y$-intercepts of $f$.

ANSWER: b. $x^2 + 6x + 5 = (x + 5)(x + 1)$ is positive on $x < -5$
and negative on $-5 < x < -1$
and positive on $-1 < x$.

(i) $\text{dom}[f] = (-\infty, -5] \cup [-1, \infty)$

(ii) $f(0) = \sqrt{5}$ is the $y$-intercept.

$x$-intercepts: $[f(x) = 0]$ iff $[x = -5 \text{ or } x = -1]$

$f$ is positive on $(-\infty, -5)$ and on $(-1, \infty)$. 

0500-16. Let \( f(x) = \sqrt{x^2 + 6x + 5} \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(iii) Find all vert/horiz asymptotes of \( f \).

\[ \frac{d}{dx} f(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 6x + 5}} \cdot (2x + 6) = \frac{x + 3}{\sqrt{x^2 + 6x + 5}}. \]

At \( x = -3 \), \( f \) has a horizontal asymptote: \( y = -3 \).

\[ \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \sqrt{x^2 + 6x + 5} = \lim_{x \to -\infty} \sqrt{x^2(1 + \frac{6}{x} + \frac{5}{x^2})} = \lim_{x \to -\infty} |x| \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = \lim_{x \to -\infty} -x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = -\infty. \]

At \( x = -3 \), \( f \) has a vertical asymptote: \( x = -3 \).

\[ \lim_{x \to -3} f(x) = \lim_{x \to -3} \sqrt{x^2 + 6x + 5} = \lim_{x \to -3} \sqrt{(x + 3)^2} = \lim_{x \to -3} |x + 3| = 0. \]

\[ \lim_{x \to -3} \frac{x + 3}{\sqrt{x^2 + 6x + 5}} = \lim_{x \to -3} \frac{x + 3}{|x + 3|} = \lim_{x \to -3} \frac{x + 3}{-(x + 3)} = -1. \]

\[ \lim_{x \to -3} (f(x) - x) = \lim_{x \to -3} \sqrt{x^2 + 6x + 5} - x = \lim_{x \to -3} \left( \sqrt{x^2 + 6x + 5} - x \right) = \lim_{x \to -3} \left( \frac{(\sqrt{x^2 + 6x + 5} - x)(\sqrt{x^2 + 6x + 5} + x)}{\sqrt{x^2 + 6x + 5} + x} \right) = \lim_{x \to -3} \frac{x^2 + 6x + 5 - x^2}{\sqrt{x^2 + 6x + 5} + x} = \lim_{x \to -3} \frac{6x + 5}{\sqrt{x^2 + 6x + 5} + x} = \lim_{x \to -3} \frac{6(\sqrt{x^2 + 6x + 5} + x)}{6x + 5} = \lim_{x \to -3} \frac{\sqrt{x^2 + 6x + 5} + x}{1} = 0. \]

\[ \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \sqrt{x^2 + 6x + 5} = \lim_{x \to -\infty} \sqrt{x^2(1 + \frac{6}{x} + \frac{5}{x^2})} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = -\infty. \]

\[ \lim_{x \to -3} \frac{x + 3}{\sqrt{x^2 + 6x + 5}} = \lim_{x \to -3} \frac{x + 3}{|x + 3|} = \lim_{x \to -3} \frac{x + 3}{-(x + 3)} = -1. \]

\[ \lim_{x \to -3} (f(x) + x) = \lim_{x \to -3} \sqrt{x^2 + 6x + 5} + x = \lim_{x \to -3} \left( \sqrt{x^2 + 6x + 5} + x \right) = \lim_{x \to -3} \left( \frac{(\sqrt{x^2 + 6x + 5} + x)(\sqrt{x^2 + 6x + 5} - x)}{\sqrt{x^2 + 6x + 5} - x} \right) = \lim_{x \to -3} \frac{x^2 + 6x + 5 - x^2}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6x + 5}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6(\sqrt{x^2 + 6x + 5} - x)}{6x + 5} = \lim_{x \to -3} \frac{\sqrt{x^2 + 6x + 5} - x}{1} = 0. \]

\[ \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \sqrt{x^2 + 6x + 5} = \lim_{x \to -\infty} \sqrt{x^2(1 + \frac{6}{x} + \frac{5}{x^2})} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = -\infty. \]

\[ \lim_{x \to -3} \frac{x + 3}{\sqrt{x^2 + 6x + 5}} = \lim_{x \to -3} \frac{x + 3}{|x + 3|} = \lim_{x \to -3} \frac{x + 3}{-(x + 3)} = -1. \]

\[ \lim_{x \to -3} (f(x) + x) = \lim_{x \to -3} \sqrt{x^2 + 6x + 5} + x = \lim_{x \to -3} \left( \sqrt{x^2 + 6x + 5} + x \right) = \lim_{x \to -3} \left( \frac{(\sqrt{x^2 + 6x + 5} + x)(\sqrt{x^2 + 6x + 5} - x)}{\sqrt{x^2 + 6x + 5} - x} \right) = \lim_{x \to -3} \frac{x^2 + 6x + 5 - x^2}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6x + 5}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6(\sqrt{x^2 + 6x + 5} - x)}{6x + 5} = \lim_{x \to -3} \frac{\sqrt{x^2 + 6x + 5} - x}{1} = 0. \]

\[ \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \sqrt{x^2 + 6x + 5} = \lim_{x \to -\infty} \sqrt{x^2(1 + \frac{6}{x} + \frac{5}{x^2})} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = \lim_{x \to -\infty} x \sqrt{1 + \frac{6}{x} + \frac{5}{x^2}} = -\infty. \]

\[ \lim_{x \to -3} \frac{x + 3}{\sqrt{x^2 + 6x + 5}} = \lim_{x \to -3} \frac{x + 3}{|x + 3|} = \lim_{x \to -3} \frac{x + 3}{-(x + 3)} = -1. \]

\[ \lim_{x \to -3} (f(x) + x) = \lim_{x \to -3} \sqrt{x^2 + 6x + 5} + x = \lim_{x \to -3} \left( \sqrt{x^2 + 6x + 5} + x \right) = \lim_{x \to -3} \left( \frac{(\sqrt{x^2 + 6x + 5} + x)(\sqrt{x^2 + 6x + 5} - x)}{\sqrt{x^2 + 6x + 5} - x} \right) = \lim_{x \to -3} \frac{x^2 + 6x + 5 - x^2}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6x + 5}{\sqrt{x^2 + 6x + 5} - x} = \lim_{x \to -3} \frac{6(\sqrt{x^2 + 6x + 5} - x)}{6x + 5} = \lim_{x \to -3} \frac{\sqrt{x^2 + 6x + 5} - x}{1} = 0. \]
0500-16. Let $f(x) = \sqrt{x^2 + 6x + 5}$.

(c) Find all max intervals of incr/decr for $f$.

**ANSWER:** $\text{dom}[f] = (-\infty, -5] \cup [-1, \infty)$

\[
f(x) = \left(x^2 + 6x + 5\right)^{1/2}
\]

\[
f'(x) = \left(1/2\right)\left(x^2 + 6x + 5\right)^{-1/2}(2x + 6)
\]

\[
= \frac{x + 3}{\sqrt{x^2 + 6x + 5}}
\]

is negative on $x < -5$

and positive on $-1 < x$.

$f$ is decreasing on $(-\infty, -5]$ and increasing on $[-1, \infty)$. 
\[ f(x) = \sqrt{x^2 + 6x + 5}. \]

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( \text{dom}[f] = (-\infty, -5] \cup [-1, \infty) \)

\[ f'(x) = \frac{x + 3}{\sqrt{x^2 + 6x + 5}} = \frac{x + 3}{(x^2 + 6x + 5)^{1/2}} \]

\[ f''(x) = \]

\[ \frac{\left[ (x^2 + 6x + 5)^{1/2} \right] [1] - [x + 3] \left[ (1/2) (x^2 + 6x + 5)^{-1/2} (2x + 6) \right]}{x^2 + 6x + 5} \]

\[ = \frac{[x^2 + 6x + 5] [1] - [x + 3] [(1/2)(2x + 6)]}{(x^2 + 6x + 5)^{3/2}} \]
0500-16. Let \( f(x) = \sqrt{x^2 + 6x + 5} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( \text{dom}[f] = (-\infty, -5] \cup [-1, \infty) \)

d. \( f''(x) = \frac{\left[x^2 + 6x + 5\right][1] - [x + 3][(1/2)(2x + 6)]}{\left(x^2 + 6x + 5\right)^{3/2}} \)

\[
= \frac{\left[x^2 + 6x + 5\right] - [x + 3][x + 3]}{\left(x^2 + 6x + 5\right)^{3/2}}
\]

\[
= \frac{\left[x^2 + 6x + 5\right] - [x^2 + 6x + 9]}{\left(x^2 + 6x + 5\right)^{3/2}}
\]
Let \( f(x) = \sqrt{x^2 + 6x + 5} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( \text{dom}[f] = (-\infty, -5] \cup [-1, \infty) \)

d. \( f''(x) = \frac{\left[ x^2 + 6x + 5 \right] - \left[ x^2 + 6x + 9 \right]}{(x^2 + 6x + 5)^{3/2}} \)

\[
= \frac{-4}{(x^2 + 6x + 5)^{3/2}}
\]

is negative on \( x < -5 \) and on \(-1 < x \).

\( f \) is concave down on \( (-\infty, -5] \) and on \([-1, \infty) \).
0500-16. Let $f(x) = \sqrt{x^2 + 6x + 5}$.

e. Sketch the graph of $f$.

**ANSWER:** e.
Let \( f(x) = x - \sin x \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-17. Let $f(x) = x - \sin x$.

a. Describe the symmetries, if any, of $f$.
b. Find all max intervals of pos/neg for $f$.

Also:
(i) What is the domain of $f$?
(ii) Find all $x$- and $y$-intercepts of $f$.
(iii) Find all vert/horiz asymptotes of $f$.

ANSWER:
a. $f$ is odd. Note: $f(x + 2\pi) = (f(x)) + 2\pi$
b. (i) $\text{dom}[f] = \mathbb{R} \supseteq [0, \infty)$
(ii) deferred until after c.
max intervals of pos/neg for $f$
also deferred until after c.
(iii) vertical/horizontal asymptotes: none
0500-17. Let \( f(x) = x - \sin x \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

c. \( f'(x) = 1 - \cos x \)

is positive on \( n\pi < x < (n + 1)\pi, \forall n \in \mathbb{Z} \).

\( f \) is increasing on \( \mathbb{R} \).

b. (ii) \( f(0) = 0 \) is the \( y \)-intercept.

So, because \( f \) is increasing on \( \mathbb{R} \),

\( f \) is positive on \( (0, \infty) \)

and \( f \) is negative on \( (-\infty, 0) \).

Then \( f(x) = 0 \) iff \( x = 0 \).

\( x \)-intercepts: 0
0500-17. Let \( f(x) = x - \sin x \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[
f'(x) = 1 - \cos x
\]

d. \( f''(x) = \sin x \)

is positive on \( 2n\pi < x < (2n + 1)\pi, \ \forall n \in \mathbb{Z} \), and negative on \( (2n - 1)\pi < x < 2n\pi, \ \forall n \in \mathbb{Z} \).

\( f \) is concave up on \( (2n\pi, (2n + 1)\pi), \ \forall n \in \mathbb{Z} \), and concave down on \( ((2n - 1)\pi, 2n\pi), \ \forall n \in \mathbb{Z} \).
0500-17. Let $f(x) = x - \sin x$.

e. Sketch the graph of $f$.

**ANSWER:**

e.
0500-18. Let \( f(x) = 2xe^{-x^2/2} \).

a. **Describe** the symmetries, if any, of \( f \).

b. **Find all** max intervals of pos/neg for \( f \).
   
   Also:
   
   (i) **What** is the domain of \( f \)?
   
   (ii) **Find all** \( x \)- and \( y \)-intercepts of \( f \).
   
   (iii) **Find all** vert/horiz asymptotes of \( f \).

c. **Find all** max intervals of incr/decr for \( f \).

d. **Find all** max intervals of cc up/cc dn for \( f \).

e. **Sketch** the graph of \( f \).
Let \( f(x) = 2xe^{-x^2/2} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

(iii) Find all vert/horiz asymptotes of \( f \).

**ANSWER:**

a. \( f \) is odd.

b. (i) \( \text{dom}[f] = \mathbb{R} \supseteq [0, \infty) \)

(ii) \( f(0) = 0 \) is the \( y \)-intercept.

\( x \)-intercepts: \([f(x) = 0] \iff [x = 0] \)

\( f \) is positive on \((0, \infty)\).

odd gives: \( f \) is negative on \((-\infty, 0)\).

(iii) vertical asymptotes: none

horizontal asymptotes: \( y = 0 \)
Let \( f(x) = 2xe^{-x^2/2} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

c. \[
f'(x) = [2] \left[ e^{-x^2/2} \right] + [2x] \left[ e^{-x^2/2} (-x) \right] 
\]

\[
= 2 \left[ 1 - x^2 \right] \left[ e^{-x^2/2} \right] 
\]

\[
= -2 \left[ (x + 1)(x - 1) \right] \left[ e^{-x^2/2} \right] 
\]

is negative on \( x < -1 \),

and positive on \( -1 < x < 1 \),

and negative on \( 1 < x \).

\( f \) is decreasing on \((-\infty, -1]\),

and increasing on \([-1, 1]\),

and decreasing on \([1, \infty)\).
0500-18. Let \( f(x) = 2xe^{-x^2/2} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:** \( f'(x) = 2 \left[ 1 - x^2 \right] e^{-x^2/2} \)

\[
f''(x) = 2 \left[ -2x \right] e^{-x^2/2} + 2 \left[ 1 - x^2 \right] \left( e^{-x^2/2} \right) (-x)
\]

\[
= 2 \left[ -2x - x + x^3 \right] e^{-x^2/2}
\]

\[
= 2 \left[ x^3 - 3x \right] e^{-x^2/2}
\]

\[
= 2 \left[ x(x^2 - 3) \right] e^{-x^2/2}
\]

\[
= 2 \left[ (x + \sqrt{3})x(x - \sqrt{3}) \right] e^{-x^2/2}
\]
0500-18. Let \( f(x) = 2xe^{-x^2/2} \).

d. Find all max intervals of cc up/cc dn for \( f \).

**ANSWER:**

\[ f''(x) = 2 \left[ (x + \sqrt{3}) x (x - \sqrt{3}) \right] \left[ e^{-x^2/2} \right] \]

is negative on \( x < -\sqrt{3} \)
and positive on \( -\sqrt{3} < x < 0 \)
and negative on \( 0 < x < \sqrt{3} \)
and positive on \( \sqrt{3} < x \).

\( f \) is concave down on \((-\infty, -\sqrt{3}]\)
and concave up on \([-\sqrt{3}, 0]\)
and concave down on \([0, \sqrt{3}]\)
and concave up on \([\sqrt{3}, \infty]\).
0500-18. Let $f(x) = 2xe^{-x^2/2}$.

e. Sketch the graph of $f$.

ANSWER:

e.
0500-19. Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \). Also:
   (i) What is the domain of \( f \)?
   (ii) Find all \( x \)- and \( y \)-intercepts of \( f \).
   (iii) Find all vert/horiz asymptotes of \( f \).

c. Find all max intervals of incr/decr for \( f \).

d. Find all max intervals of cc up/cc dn for \( f \).

e. Sketch the graph of \( f \).
0500-19. Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

a. Describe the symmetries, if any, of \( f \).

b. Find all max intervals of pos/neg for \( f \).

Also:

(i) What is the domain of \( f \)?

(ii) Find all \( x \)- and \( y \)-intercepts of \( f \).

**ANSWER:**

a. NO symmetries

b. \( 3^2 - 4(1)(4) < 0 \), so \( x^2 + 3x + 4 \) has no real roots.

\[ \forall x \in \mathbb{R}, \quad x^2 + 3x + 4 > 0 \]

(i) \( \text{dom}[f] = \mathbb{R} \setminus \{-3\} \)

(ii) \( f(0) = 4/3 \) is the \( y \)-intercept.

\[ \forall x \in \text{dom}[f], \ f(x) \neq 0, \text{ so no } x \text{-intercepts} \]

\( f(x) \) is negative on \( x < -3 \) and positive on \( -3 < x \).
0500-19. Let $f(x) = \frac{x^2 + 3x + 4}{x + 3}$.

b. Find all max intervals of pos/neg for $f$. Also:
   (iii) Find all vert/horiz asymptotes of $f$.

ANSWER: $f(x)$ is negative on $x < -3$ and positive on $-3 < x$.

b. (iii)

$$\lim_{x \to -\infty} f(x) = -\infty \quad \lim_{x \to \infty} f(x) = \infty$$

$f$ has no horizontal asymptotes.

$$\lim_{x \to -3^-} f(x) = -\infty \quad \lim_{x \to -3^+} f(x) = \infty$$

$x = -3$ is the only vertical asymptote for $f$. 
Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

\[
f'(x) = \frac{[x + 3][2x + 3] - [x^2 + 3x + 4][1]}{(x + 3)^2}
\]

\[
= \frac{[2x^2 + 9x + 9] - [x^2 + 3x + 4]}{(x + 3)^2}
\]

\[
= \frac{x^2 + 6x + 5}{(x + 3)^2} = \frac{(x + 5)(x + 1)}{(x + 3)^2}
\]
0500-19. Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

c. Find all max intervals of incr/decr for \( f \).

**ANSWER:**

\[
\text{dom}[f] = \mathbb{R} \setminus \{-3\}
\]

\[
f'(x) = \frac{x^2 + 6x + 5}{(x + 3)^2} = \frac{(x + 5)(x + 1)}{(x + 3)^2}
\]

is positive on \( x < -5 \)

and negative on \( -5 < x < -3 \)

and negative on \( -3 < x < -1 \)

and positive on \( -1 < x \).

\( f \) is increasing on \( (-\infty, -5] \)
and decreasing on \( [-5, -3) \)
and decreasing on \( (-3, -1] \)
and increasing on \( [-1, \infty) \).
Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

**d. Find all max intervals of cc up/cc dn for \( f \).**

**ANSWER:** \( f'(x) = \frac{x^2 + 6x + 5}{(x + 3)^2} \)

\[
f''(x) = \frac{[(x + 3)^2][2x + 6] - [x^2 + 6x + 5][2(x + 3)][1]}{(x + 3)^3}
\]

\[
= \frac{[x + 3][2x + 6] - [x^2 + 6x + 5][2]}{(x + 3)^3}
\]

\[
= \frac{2x^2 + 12x + 18 - [2x^2 + 12x + 10]}{(x + 3)^3}
\]

\[
= \frac{8}{(x + 3)^3}
\]
Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

\[ f'(x) = \frac{x^2 + 6x + 5}{(x + 3)^2} \quad \text{dom}[f] = \mathbb{R} \setminus \{-3\} \]

\[ f''(x) = \frac{8}{(x + 3)^3} \]

is negative on \( x < -3 \)
and positive on \( -3 < x \).

\( f \) is concave down on \( (-\infty, -3) \)
and concave up on \( (-3, \infty) \).
0500-19. Let \( f(x) = \frac{x^2 + 3x + 4}{x + 3} \).

e. Sketch the graph of \( f \).

**ANSWER:**

e.