CALCULUS
Definite integration and Riemann sum problems
OLD
0590-1. Let $f(x) = 4 - x^2$.

a. Compute $L_4S_{-2}^2f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $L_4S_{-2}^2f$.

b. Compute $M_4S_{-2}^2f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $M_4S_{-2}^2f$.

c. Compute $R_4S_{-2}^2f$.
   Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $R_4S_{-2}^2f$. 

0590-1. Let \( f(x) = 4 - x^2 \).

a. Compute \( L_4 S^2_{-2} f \).

Sketch \( f \) over \([-2, 2]\) and add, into your sketch, the four rectangles represented by \( L_4 S^2_{-2} f \).

ANS:

\[
L_4 S^2_{-2} f = 0 + 3 + 4 + 3 = 10
\]
0590-1. Let $f(x) = 4 - x^2$.

b. Compute $M_4S^2_{-2}f$.

Sketch $f$ over $[-2, 2]$ and add, into your sketch, the four rectangles represented by $M_4S^2_{-2}f$.

ANS: 

$$M_4S^2_{-2}f = 1.75 + 3.75 + 3.75 + 1.75 = 11$$
0590-1. Let \( f(x) = 4 - x^2 \).

c. Compute \( R_4 S^2_{-2} f \).

Sketch \( f \) over \([-2, 2]\) and add, into your sketch, the four rectangles represented by \( R_4 S^2_{-2} f \).

ANS:

c.

\[
R_4 S^2_{-2} f = 3 + 4 + 3 + 0 = 10
\]
0590-2. Let \( f(x) = e^x + 5 \).

a. Compute \( L_3 S^6_0 f \) to three decimal places.

\[
L_3 S^6_0 f = 2[(e^0 + 5) + (e^2 + 5) + (e^4 + 5)]
\]
\[
= 2[1 + e^2 + e^4 + 15] \approx 155.974
\]

b. Compute \( M_3 S^6_0 f \) to three decimal places.

\[
M_3 S^6_0 f = 2[(e^1 + 5) + (e^3 + 5) + (e^5 + 5)]
\]
\[
= 2[e + e^3 + e^5 + 15] \approx 372.434
\]

c. Compute \( R_3 S^6_0 f \) to three decimal places.

\[
R_3 S^6_0 f = 2[(e^2 + 5) + (e^4 + 5) + (e^6 + 5)]
\]
\[
= 2[e^2 + e^4 + e^6 + 15] \approx 960.832
\]
0590-3. Let $f(x) = \sin^2 x$.

a. Compute $L_5 S_0^\pi f$ to three decimal places.
b. Compute $M_5 S_0^\pi f$ to three decimal places.
c. Compute $R_5 S_0^\pi f$ to three decimal places.
0590-3. Let \( f(x) = \sin^2 x \).

a. Compute \( L_5 S^{\pi}_0 f \) to three decimal places.

**ANSWER:**

\[
L_5 S^{\pi}_0 f = \frac{\pi}{5} \left[ (\sin^2 (0)) + (\sin^2 \left( \frac{\pi}{5} \right)) + (\sin^2 \left( \frac{2\pi}{5} \right)) + (\sin^2 \left( \frac{3\pi}{5} \right)) + (\sin^2 \left( \frac{4\pi}{5} \right)) \right]
\]

\[
= [0.628] [0 + 0.345 + 0.905 + 0.905 + 0.345]
\]

\[
= 1.571
\]

**NOTE:** \( \frac{\pi}{2} \approx 1.571 \)
0590-3. Let $f(x) = \sin^2 x$.

b. Compute $M_5 S_0^\pi f$ to three decimal places.

**Answer:**

$$b. \, M_5 S_0^\pi f = \frac{\pi}{5} \left[ (\sin^2 \left( \frac{\pi}{10} \right)) + (\sin^2 \left( \frac{3\pi}{10} \right)) + (\sin^2 \left( \frac{5\pi}{10} \right)) + (\sin^2 \left( \frac{7\pi}{10} \right)) + (\sin^2 \left( \frac{9\pi}{10} \right)) \right]$$

$$\approx [0.628][0.095 + 0.655 + 1 + 0.655 + 0.095]$$

$$\approx 1.571$$

**Note:** $\frac{\pi}{2} \approx 1.571$
0590-3. Let \( f(x) = \sin^2 x \).

\[ \text{c. Compute } R_5 S_0^\pi f \text{ to three decimal places.} \]

**ANSWER:**

\[ \text{c. } R_5 S_0^\pi f = \frac{\pi}{5} \left[ (\sin^2 \left( \frac{\pi}{5} \right)) + (\sin^2 \left( \frac{2\pi}{5} \right)) + (\sin^2 \left( \frac{3\pi}{5} \right)) + (\sin^2 \left( \frac{4\pi}{5} \right)) + (\sin^2 (\pi)) \right] \]

\[ = [0.628] [0.345 + 0.905 + 0.905 + 0.345 + 0] \]

\[ \hat{=} 1.571 \]

**NOTE:** \( \frac{\pi}{2} \hat{=} 1.571 \)
A car’s acceleration is positive from time 0 to time 12 seconds, and its velocity at various times is given in the table below.

<table>
<thead>
<tr>
<th>time (secs)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocity (ft/sec)</td>
<td>0</td>
<td>20</td>
<td>35</td>
<td>45</td>
<td>53</td>
<td>59</td>
<td>62</td>
</tr>
</tbody>
</table>

Find upper and lower estimates for the distance traveled by the car over these 12 seconds.

**ANSWER:**

upper estimate = $[2][20+35+45+53+59+62]$

$= [2][274] = 548$ ft

lower estimate = $[2][0+20+35+45+53+59]$

$= [2][212] = 424$ ft
The gph of a function $f$ appears below.

Estimate $\int_{0}^{10} f(x) \, dx$ by computing

(a) $L_5 S_0^{10} f$,  \hspace{1cm} (b) $M_5 S_0^{10} f$

and (c) $R_5 S_0^{10} f$.\[16\]
The gph of a function $f$ appears below.

\[
\text{ANSWER: } \\
(a) \ L_5 S_0^{10} f = 2[4 + (-1) + (-2) + 2 + 3] \\
= 12
\]
The gph of a function $f$ appears below.

**ANSWER:**

(b) $M_5 S_0^{10} f = 2[1 + (-2) + 0 + 3 + 2] = 8$
The gph of a function $f$ appears below.

**ANSWER:**

(c) $R_5 S_0^{10} f = 2[(-1) + (-2) + 2 + 3 + 1]$

= 6
0590-6. Express the area under \( y = e^{-x^2} \) from \( x = -1 \) to \( x = 1 \) as a limit of midpoint Riemann sums. (Don’t evaluate the limit.)

\[
\lim_{n \to \infty} \frac{2}{n} \left[ \sum_{j=1}^{n} e^{-\left[-1+j\left(\frac{2}{n}\right)-(1/n)\right]^2} \right]
\]
Express the area under $y = \sqrt{x^3 + x + 5}$ from $x = 2$ to $x = 4$ as a limit of left endpoint Riemann sums. (Don’t evaluate the limit.)

ANSWER:

$$\lim_{n \to \infty} \frac{2}{n} \sum_{j=0}^{n-1} \sqrt{(2 + j(2/n))^3 + (2 + j(2/n)) + 5}$$
Express the area under \( y = \cos(x^3) \) from \( x = -2 \) to \( x = 5 \) as a limit of right endpt Riemann sums. (Don’t evaluate the limit.)

**ANSWER:**

\[
\lim_{{n \to \infty}} \frac{7}{n} \left[ \sum_{j=1}^{n} \cos\left((-2 + j(7/n))^3\right) \right]
\]
0590-9. Express \( \int_5^{13} \frac{e^{-x^2/2}}{\sqrt{2\pi}} \, dx \) as a limit of midpoint Riemann sums. 

(Don’t evaluate the limit.)

**ANSWER:**

\[
\lim_{n \to \infty} \frac{8}{n} \left[ \sum_{j=1}^{n} \frac{e^{-[5+j(8/n)-(4/n)]^2/2}}{\sqrt{2\pi}} \right]
\]
0590-10. Let $f(x) = x^3$.

a. Write $R_nS_0^2f$ as a rational expression in $n$ (i.e., as one polynomial in $n$ divided by another).

b. Compute $\lim_{n \to \infty} R_nS_0^2f$. 
Let \( f(x) = x^3 \).

a. Write \( R_n S_0^2 f \) as a rational expression in \( n \) (i.e., as one polynomial in \( n \) divided by another).

**ANSWER:**

\[
R_n S_0^2 f = \frac{2}{n} \left[ \sum_{j=1}^{n} \left( 0 + j \left( \frac{2}{n} \right) \right)^3 \right]
\]

\[
= \frac{2}{n} \left[ \sum_{j=1}^{n} j^3 \left( \frac{8}{n^3} \right) \right]
\]

\[
= \frac{16}{n^4} \left[ \sum_{j=1}^{n} j^3 \right]
\]
Let \( f(x) = x^3 \).

a. Write \( R_nS_0^2f \) as a rational expression in \( n \) (i.e., as one polynomial in \( n \) divided by another).

ANSWER: a. \[
R_nS_0^2f = \frac{16}{n^4} \left[ \sum_{j=1}^{n} j^3 \right]
\]

\[
= \frac{16}{n^4} \left[ \frac{n^4(n+1)^2}{4} \right]
\]

\[
= \frac{4(n+1)^2}{n^2}
\]
Let $f(x) = x^3$.

a. Write $R_nS_0^2 f$ as a rational expression in $n$ (i.e., as one polynomial in $n$ divided by another).

**ANSWER:**

$$R_nS_0^2 f = \frac{4(n + 1)^2}{n^2}$$

$$= \frac{4(n^2 + 2n + 1)}{n^2}$$

$$= \frac{4n^2 + 8n + 4}{n^2}$$
0590-10. Let $f(x) = x^3$.

b. Compute $\lim_{n \to \infty} R_n S_0^2 f$.

**ANSWER: a.**

\[
R_n S_0^2 f = \frac{4n^2 + 8n + 4}{n^2}
\]

**ANSWER: b.**

\[
\lim_{n \to \infty} R_n S_0^2 f = \lim_{n \to \infty} \frac{4n^2 + 8n + 4}{n^2} = 4
\]
0590-11. The limit

\[
\lim_{n \to \infty} \left[ \frac{4}{n} \sum_{j=1}^{n} \left( e^{\sin(3+(4/n)j)} \right) \right]
\]

represents the area under \( y = f(x) \)
from \( x = a \) to \( x = b \),
for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit
as a definite integral.

a. \( f(x) = e^{\sin x}, a = 3, b = 7 \)

b. \( \int_{3}^{7} e^{\sin x} \, dx \)
The limit
\[
\lim_{n \to \infty} \left[ \frac{7}{n} \sum_{j=0}^{n-1} \cos \left( e^{2 + (7/n)j} \right) \right]
\]
represents the area under \( y = f(x) \) from \( x = a \) to \( x = b \), for some choice of \( f(x) \), \( a \) and \( b \).

a. Find \( f(x) \), \( a \) and \( b \).

b. Express the limit as a definite integral.

a. \( f(x) = \cos(e^x) \), \( a = 2 \), \( b = 9 \)

b. \( \int_{2}^{9} \cos(e^x) \, dx \)
Let \( f(x) = 3 + \sqrt{4 - x^2} \).

a. Sketch the graph of \( y = f(x) \).

b. Compute \( \int_{-2}^{2} f(x) \, dx \), by interpreting this integral as an area.

**ANSWER:**

\[
\begin{align*}
\text{a. } & \quad \frac{\pi \cdot 2^2}{2} + 3 \cdot 4 \\
\text{b. } & \quad \frac{\pi \cdot 2^2}{2} + 3 \cdot 4 \\
& \quad = 2\pi + 12
\end{align*}
\]