CALCULUS
Volume by slices and the disk and washer methods:
Problems
0720-1. Let \( R \) be the region bounded by \( y = x + 1 \) and \( x = 2 \) in \( 1 \leq y \leq 2 \).

a. Sketch \( R \).
b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

ANSWER:

a. \( y \)
\( x = 2 \)
\( y = x + 1 \)
\( R \)
\( y = 2 \)
\( y = 1 \)
\( x \)
0720-1. Let \( R \) be the region bounded by 
\[ y = x + 1 \text{ and } x = 2 \text{ in } 1 \leq y \leq 2. \]

a. Sketch \( R \).
b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

b. 
\[
\int_0^1 [\pi (x + 1)^2 - \pi (1^2)] \, dx
\] 
continued on next slide
0720-1. Let $R$ be the region bounded by $y = x + 1$ and $x = 2$ in $1 \leq y \leq 2$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

b. $\int_0^1 \left[ \pi(x + 1)^2 - \pi(1^2) \right] \, dx + \int_1^2 \pi(2^2) - \pi(1^2) \, dx$
0720-1. Let $R$ be the region bounded by $y = x + 1$ and $x = 2$ in $1 \leq y \leq 2$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

b. $y = x + 1$

\[
\int_0^1 \left[ \pi(x + 1)^2 - \pi(1^2) \right] \, dx + \int_1^2 \pi(2^2) - \pi(1^2) \, dx
\]

\[
= \pi \left[ \int_0^1 x^2 + 2x \, dx \right] + \pi \left[ \int_1^2 3 \, dx \right]
\]

\[
= \pi \left[ \frac{1^3}{3} + 1^2 \right] + \pi [3]
\]

\[
= \pi \left[ \frac{1}{3} + 1 + 3 \right] = \frac{13\pi}{3}
\]
0720-1. Let \( R \) be the region bounded by 
\[
y = x + 1 \quad \text{and} \quad x = 2 \quad \text{in} \quad 1 \leq y \leq 2.
\]

a. Sketch \( R \).
b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

\[
\int_1^2 \left[ \pi(2^2) - \pi(y - 1)^2 \right] dy
\]

\[
= \pi \int_1^2 4 - (y^2 - 2y + 1) \, dy
\]

\[
= \pi \int_1^2 -y^2 + 2y + 3 \, dy
\]

\[
= \pi \left[ -\frac{2^3 - 1^3}{3} + (2^2 - 1^2) + 3(2 - 1) \right]
\]

\[
= \pi \left[ -\frac{7}{3} + 6 \right] = \frac{11\pi}{3}
\]
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \]  
and \( y = x \).

\begin{enumerate}[a.]
\item Sketch \( R \).
\item Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.
\item Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.
\end{enumerate}
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \] and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

b. 
\[ y - 1 = (x - 1)^2 \]
\[ \int_1^2 \pi x^2 - \pi \left(1 + (x - 1)^2\right)^2 \, dx \]
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \] and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

\[
\int_1^2 \pi x^2 - \pi \left( 1 + (x - 1)^2 \right)^2 \, dx
\]

\[
= \pi \int_1^2 x^2 - (x^2 - 2x + 2)^2 \, dx
\]

\[
= \pi \int_1^2 x^2 - (x^4 - 4x^3 + 8x^2 - 8x + 4) \, dx
\]

\[
= \pi \int_1^2 -x^4 + 4x^3 - 7x^2 + 8x - 4 \, dx
\]

\[
= \pi \left[ -\frac{31}{5} + 15 - \frac{49}{3} + 12 - 4 \right]
\]

\[
= \pi \left[ -\frac{93}{15} + 23 - \frac{245}{15} \right]
\]
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \] and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

b. 
\[
\int_1^2 \pi x^2 - \pi \left(1 + (x - 1)^2\right)^2 \, dx
\]
\[
= \pi \left[ -\frac{93}{15} + 23 - \frac{245}{15} \right]
\]
\[
= \pi \left[ -\frac{93}{15} + \frac{345}{15} - \frac{245}{15} \right]
\]
\[
= \frac{7\pi}{15}
\]
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \]  and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

\[ \int_1^2 \pi \left( 1 + \sqrt{y-1} \right)^2 - \pi \left( y^2 \right) \, dy \]
0720-2. Let \( R \) be the region bounded by \( y - 1 = (x - 1)^2 \) and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

C. \[ y - 1 = (x - 1)^2 \]

\[
\int_1^2 \pi \left( 1 + \sqrt{y - 1} \right)^2 - \pi (y^2) \, dy
\]

\[
= \pi \int_1^2 (1 + 2\sqrt{y - 1} + y - 1) - y^2 \, dy
\]

\[
= \pi \int_1^2 -y^2 + y + 2(y - 1)^{1/2} \, dy
\]
Let $R$ be the region bounded by 
$y - 1 = (x - 1)^2$ and $y = x$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

C. 

\[
\int_1^2 \pi \left( 1 + \sqrt{y - 1} \right)^2 - \pi (y^2) \, dy
\]

\[
= \pi \int_1^2 -y^2 + y + 2(y - 1)^{1/2} \, dy
\]

\[
= \pi \left[ -\frac{y^3}{3} + \frac{y^2}{2} + \frac{2(y - 1)^{3/2}}{3/2} \right]_{y \to 1}^{y \to 2}
\]

\[
= \pi \left[ -\frac{2^3}{3} + \frac{2^2}{2} + \frac{2(2 - 1)^{3/2}}{3/2} - \left( -\frac{1^3}{3} + \frac{1^2}{2} + \frac{2(1 - 1)^{3/2}}{3/2} \right) \right]
\]

\[
= \pi \left[ \frac{8}{3} - \frac{1}{2} + \frac{2}{3} - \frac{1}{3} \right]
\]

\[
= \pi \left( \frac{7}{3} \right)
\]
0720-2. Let $R$ be the region bounded by $y - 1 = (x - 1)^2$ and $y = x$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

C. $y - 1 = (x - 1)^2$

\[
\int_{1}^{2} \pi \left( 1 + \sqrt{y-1} \right)^2 - \pi \left( y^2 \right) \, dy
\]

\[
= \pi \left[ -\frac{y^3}{3} + \frac{y^2}{2} + \frac{2(y-1)^{3/2}}{3/2} \right]_{1}^{2}
\]

\[
= \pi \left[ -\frac{2^3 - 1^3}{3} + \frac{2^2 - 1^2}{2} + \frac{2[(1)^{3/2} - 0]}{3/2} \right]
\]
0720-2. Let \( R \) be the region bounded by 
\[ y - 1 = (x - 1)^2 \] and \( y = x \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

c. Find the volume of the solid obtained by rotating \( R \) about the \( y \)-axis.

**ANSWER:**

C.

\[ y - 1 = (x - 1)^2 \]

\[
\int_1^2 \pi \left(1 + \sqrt{y - 1}\right)^2 - \pi \left(y^2\right) \, dy
\]

\[
= \pi \left[ -\frac{2^3 - 1^3}{3} + \frac{2^2 - 1^2}{2} + \frac{2[(1)^{3/2} - 0]}{3/2} \right]
\]

\[
= \pi \left[ -\frac{7}{3} + \frac{3}{2} + \frac{2}{3/2} \right]
\]

\[
= \pi \left[ -\frac{7}{3} + \frac{3}{2} + \frac{4}{3} \right]
\]
0720-2. Let $R$ be the region bounded by $y - 1 = (x - 1)^2$ and $y = x$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

c. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

C. 

\[
\int_1^2 \pi \left( 1 + \sqrt{y - 1} \right)^2 - \pi (y^2) \, dy
\]

\[
= \pi \left[ -\frac{7}{3} + \frac{3}{2} + \frac{4}{3} \right]
\]

\[
= \pi \left[ -1 + \frac{3}{2} \right] = \frac{\pi}{2}
\]
Let $R$ be the region bounded by $y = \ln x$, $x = 4$ and $y = 1$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $y$-axis.

**ANSWER:**

b. 
\[
\int_1^{\ln 4} \pi (4^2) - \pi (e^y)^2 \, dy
= \pi \int_1^{\ln 4} 16 - e^{2y} \, dy
= \pi \left[ 16y - \frac{e^{2y}}{2} \right]_{y = 1}^{y = \ln 4}
= \pi \left[ 16 \ln 4 - \frac{(e^{\ln 4})^2}{2} - 16 + \frac{e^2}{2} \right]
= \pi \left[ 16 \ln 4 + \frac{e^2}{2} - 24 \right]
\]
0720-4. Let $R$ be the region bounded by $y = \sin x$ and $y = 0$ in $0 \leq x \leq \pi$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**Hint:** $\sin^2 x = \frac{1 - [\cos(2x)]}{2}$

**Answer:**

a. 

b. 

\[
\int_0^\pi \pi (\sin x)^2 \, dx = \pi \int_0^\pi \sin^2 x \, dx = \pi \int_0^\pi \frac{1 - [\cos(2x)]}{2} \, dx = \pi \left[ \frac{x - [(\sin(2x))/2]}{2} \right]_0^\pi = \frac{\pi^2}{2} \]
Let $R$ be the region bounded by 
$$x^2 + (y - 3)^2 = 1.$$ 

a. Sketch $R$. 
b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

Note: This solid is called a torus. It is in the shape of a doughnut.

Hint: Remember that 
$$2 \int_{-1}^{1} \sqrt{1 - x^2} \, dx$$ 
is known; it is the area enclosed in a circle of radius 1.
0720-5. Let $R$ be the region bounded by $x^2 + (y - 3)^2 = 1$.

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**Hint:** Remember that $2 \int_{-1}^{1} \sqrt{1 - x^2} \, dx$ is known; it is the area enclosed in a circle of radius 1.

**ANSWER:**

a. $x^2 + (y - 3)^2 = 1$

b. $\int_{-1}^{1} \pi \left( 3 + \sqrt{1 - x^2} \right)^2 - \pi \left( 3 - \sqrt{1 - x^2} \right)^2 \, dx$

$$= \pi \int_{-1}^{1} \left( 9 + 6\sqrt{1 - x^2} + 1 - x^2 \right) - \left( 9 - 6\sqrt{1 - x^2} + 1 - x^2 \right) \, dx$$

$$= \pi \int_{-1}^{1} 12\sqrt{1 - x^2} \, dx$$
0720-5. Let $R$ be the region bounded by 
\[ x^2 + (y - 3)^2 = 1. \]

a. Sketch $R$.

b. Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**Hint:** Remember that $2 \int_{-1}^{1} \sqrt{1 - x^2} \, dx$ is known; it is the area enclosed in a circle of radius 1.

**ANSWER:**

\[ \begin{align*}
\text{a. } & \quad x^2 + (y - 3)^2 = 1 \\
\text{b. } & \quad \int_{-1}^{1} \pi \left( 3 + \sqrt{1 - x^2} \right)^2 - \pi \left( 3 - \sqrt{1 - x^2} \right)^2 \, dx \\
& \quad = \pi \int_{-1}^{1} 12\sqrt{1 - x^2} \, dx \\
& \quad = 6\pi \left[ 2 \int_{-1}^{1} \sqrt{1 - x^2} \, dx \right] \\
& \quad = 6\pi \left[ \pi \cdot 1^2 \right] = 6\pi^2 
\end{align*} \]
0720-6. Let \( R \) be the region bounded by \( y = x^3 \) and \( x = y^4 \).

a. Sketch \( R \).

b. Find the volume of the solid obtained by rotating \( R \) about the line \( y = -1/2 \).

c. Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).
0720-6. Let $R$ be the region bounded by $y = x^3$ and $x = y^4$.

a. Sketch $R$.

**ANSWER:**

a.
0720-6. Let $R$ be the region bounded by 
\[ y = x^3 \text{ and } x = y^4. \]

b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.

**ANSWER:**

\[ \int_0^1 \pi \left( x^{1/4} + \frac{1}{2} \right)^2 - \pi \left( x^3 + \frac{1}{2} \right)^2 \, dx \]

\[ = \pi \int_0^1 (x^{1/2} + x^{1/4} + \frac{1}{4}) - (x^6 + x^3 + \frac{1}{4}) \, dx \]

\[ = \pi \int_0^1 -x^6 - x^3 + x^{1/2} + x^{1/4} \, dx \]

\[ = \pi \left[ -\frac{x^7}{7} - \frac{x^4}{4} + \frac{x^{3/2}}{3/2} + \frac{x^{5/4}}{5/4} \right]_{x=0}^{x=1} \]
Let $R$ be the region bounded by $y = x^3$ and $x = y^4$.

b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.

**ANSWER:**

\[
\begin{align*}
\int_0^1 \pi \left( x^{1/4} + \frac{1}{2} \right)^2 - \pi \left( x^3 + \frac{1}{2} \right)^2 \, dx \\
= \pi \left[ \frac{x^7}{7} - \frac{x^4}{4} + \frac{x^{3/2}}{3/2} + \frac{x^{5/4}}{5/4} \right]_{x:0}^{x:1} \\
= \pi \left[ \left( \frac{1}{7} - \frac{1}{4} + \frac{1}{3/2} + \frac{1}{5/4} \right) - \left[ \frac{1}{7} - \frac{1}{4} + \frac{2}{3} + \frac{4}{5} \right] \right] \\
= \pi \left[ \frac{60}{420} - \frac{105}{420} + \frac{280}{420} + \frac{336}{420} \right] \\
= \frac{451\pi}{420}
\end{align*}
\]
0720-6. Let \( R \) be the region bounded by 
\[ y = x^3 \] and \( x = y^{4/3} \).

c. Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).

\[ \begin{align*}
\text{ANSWER:} & \quad \int_0^1 \pi \left( y^{1/3} + \frac{1}{3} \right)^2 - \pi \left( y^{4/3} + \frac{1}{3} \right)^2 \, dy \\
& \quad = \pi \int_0^1 \left( y^{2/3} + \frac{2y^{1/3}}{3} + \frac{1}{9} \right) - \left( y^{8/3} + \frac{2y^{4/3}}{3} + \frac{1}{9} \right) \, dy \\
& \quad = \pi \int_0^1 -y^{8/3} - \frac{2y^{4/3}}{3} + y^{2/3} + \frac{2y^{1/3}}{3} \, dy \\
& \quad = \pi \left[ \frac{y^{9/3}}{9} - \frac{2y^{5/3}}{15} + \frac{y^{5/3}}{5/3} + \frac{2y^{4/3}}{12/3} \right]_{y=0}^{y=1} \\
& \quad = \pi \left[ \left[ -\frac{1}{9} - \frac{2}{15} + \frac{1}{5/3} + \frac{2}{4} \right] - [0] \right]^{31}
\end{align*} \]
0720-6. Let \( R \) be the region bounded by 
\[ y = x^3 \] and \( x = y^{4/3} \).

(c) Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).

**Answer:**

\[
\int_{0}^{1} \pi \left( y^{1/3} + \frac{1}{3} \right)^2 - \pi \left( y^{4/3} + \frac{1}{3} \right)^2 \, dy
\]

\[
= \pi \left[ \left( -\frac{1}{9} - \frac{2}{15} + \frac{1}{5/3} + \frac{2}{4} \right) - [0] \right]
\]

\[
= \pi \left[ -\frac{1}{9} - \frac{2}{15} + \frac{3}{5} + \frac{1}{2} \right]
\]

\[
= \frac{77\pi}{90}
\]
0720-7. Let $R$ be the region bounded by $y = x^2$ and $x = y^6$.

a. Sketch $R$.
b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.
c. Find the volume of the solid obtained by rotating $R$ about the line $x = -1/3$. 
Let $R$ be the region bounded by $y = x^2$ and $x = y^6$.

a. Sketch $R$. 

\[ \text{ANSWER:} \]

\[ a. \]

\[ y = x^2 \]

\[ x = y^6 \]
0720-7. Let $R$ be the region bounded by $y = x^2$ and $x = y^{6}$.

b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.

\[
\begin{align*}
&\quad \int_0^1 \pi \left( x^{1/6} + \frac{1}{2} \right)^2 - \pi \left( x^2 + \frac{1}{2} \right)^2 \, dx \\
&= \pi \int_0^1 \left( x^{1/3} + x^{1/6} + \frac{1}{4} \right) \\
&\quad - \left( x^4 + x^2 + \frac{1}{4} \right) \, dx \\
&= \pi \int_0^1 -x^4 - x^2 + x^{1/3} + x^{1/6} \, dx \\
&= \pi \left[ -\frac{x^5}{5} - \frac{x^3}{3} + \frac{x^{4/3}}{4/3} + \frac{x^{7/6}}{7/6} \right]_{x \to 0}^{x \to 1}
\end{align*}
\]
Let $R$ be the region bounded by $y = x^2$ and $x = y^{6}$.

b. Find the volume of the solid obtained by rotating $R$ about the line $y = -1/2$.

**Answer:**

$$
\int_0^1 \pi \left( x^{1/6} + \frac{1}{2} \right)^2 - \pi \left( x^2 + \frac{1}{2} \right)^2 \, dx
$$

$$
= \pi \left[ -\frac{x^5}{5} - \frac{x^3}{3} + \frac{x^{4/3}}{4/3} + \frac{x^{7/6}}{7/6} \right]_{x: \to 1} - [0]
$$

$$
= \pi \left[ -\frac{1}{5} - \frac{1}{3} + \frac{1}{4/3} + \frac{1}{7/6} \right]
$$

$$
= \pi \left[ -\frac{84}{420} - \frac{140}{420} + \frac{315}{420} + \frac{360}{420} \right]
$$

$$
= \frac{451\pi}{420}
$$
0720-7. Let \( R \) be the region bounded by 
\[ y = x^2 \text{ and } x = y^6. \]

\[ \text{c. Find the volume of the solid obtained by} \]
\[ \text{rotating } R \text{ about the line } x = -1/3. \]

\[ \text{ANSWER:} \]

\[ \int_0^1 \pi \left( y^{1/2} + \frac{1}{3} \right)^2 - \pi \left( y^6 + \frac{1}{3} \right)^2 \, dy \]
\[ = \pi \int_0^1 \left( y + \frac{2y^{1/2}}{3} + \frac{1}{9} \right) - \left( y^{12} + \frac{2y^6}{3} + \frac{1}{9} \right) \, dy \]
\[ = \pi \int_0^1 -y^{12} - \frac{2y^6}{3} + y + \frac{2y^{1/2}}{3} \, dy \]
\[ = \pi \left[ -\frac{y^{13}}{13} - \frac{2y^7}{21} + \frac{y^2}{2} + \frac{2y^{3/2}}{9/2} \right]_{y \to 0}^{y \to 1} \]
0720-7. Let $R$ be the region bounded by $y = x^2$ and $x = y^6$.

c. Find the volume of the solid obtained by rotating $R$ about the line $x = -1/3$.

\begin{align*}
\text{ANSWER:} & \quad \int_0^1 \pi \left( y^{1/2} + \frac{1}{3} \right)^2 - \pi \left( y^6 + \frac{1}{3} \right)^2 \, dy \\
& = \pi \left[ -\frac{y^{13}}{13} - \frac{2y^7}{21} + \frac{y^2}{2} + \frac{2y^{3/2}}{9/2} \right]_{y: \to 1}^{y: \to 0} \\
& = \pi \left[ -\frac{1}{13} - \frac{2}{21} + \frac{1}{2} + \frac{2}{9/2} \right] - [0] \\
& = \pi \left[ -\frac{1}{13} - \frac{2}{21} + \frac{1}{2} + \frac{4}{9} \right]
\end{align*}
Let \( R \) be the region bounded by \( y = x^2 \) and \( x = y^6 \).

c. Find the volume of the solid obtained by rotating \( R \) about the line \( x = -1/3 \).

\[
\int_0^1 \pi \left( y^{1/2} + \frac{1}{3} \right)^2 - \pi \left( y^6 + \frac{1}{3} \right)^2 \, dy
\]

\[= \pi \left[ -\frac{1}{13} - \frac{2}{21} + \frac{1}{2} + \frac{4}{9} \right] \]

\[= \pi \left[ -\frac{126}{1638} - \frac{156}{1638} + \frac{819}{1638} + \frac{728}{1638} \right] \]

\[= \frac{1265\pi}{1638} \]
Let $R$ be the region bounded by $y = 4 \cos x, y = e^x$ in $0 \leq x \leq \pi/4$. Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating $R$ about the line $y = 5$.

**ANSWER:**

$\forall x \in [0, \pi/4],$

\[ 5 > 4 \cos x \geq 4 \cos(\pi/4) > e^{\pi/4} \geq e^x \]

\[
\int_0^{\pi/4} \pi (5 - e^x)^2 - \pi (5 - 4 \cos x)^2 \, dx
\]
Describe the solid of revolution whose volume is given by

\[ \pi \int_1^2 \left( 9e^{8x} - 4e^{2x} \right) dx. \]

Do not evaluate this integral.

**ANSWER:**
This is the solid obtained by revolving the region bounded by

\[ y = 3e^{4x}, \quad y = 2e^x \quad \text{in} \quad 1 \leq x \leq 2 \]

about the \( x \)-axis.
Describe the solid of revolution whose volume is given by

\[ \pi \int_{\pi/2}^{\pi} (2 + \sin x)^2 - 4 \, dx. \]

Do not evaluate this integral.

**ANSWER:**
This is the solid obtained by revolving the region bounded by

\[ y = 2 + \sin x, \quad y = 2 \quad \text{in} \quad \pi/2 \leq x \leq \pi \]

about the \( x \)-axis.

**ALTERNATE ANSWER:**
This is the solid obtained by revolving the region bounded by

\[ y = \sin x, \quad y = 0 \quad \text{in} \quad \pi/2 \leq x \leq \pi \]

about the line \( y = -2 \).
A solid $S$ sits above a horizontal plane $P$. \( \forall x \geq 0, \) let \( P_x \) be the horizontal plane that is \( x \) units above \( P \). Suppose that $S$ lies between $P_1$ and $P_2$. Suppose, also, that \( \forall x \in [1, 2] \), the intersection of $S$ and $P_x$ is the region inside an ellipse whose major axis has length $x$ and whose minor axis has length $e^{2x^2}$.

**Compute** the volume of $S$.

**Hint:** Remember that if $a$ and $b$ are the major and minor axes of an ellipse $E$, then the area inside $E$ is $\pi ab/4$. 
A solid \( S \) sits above a horizontal plane \( P \). \( \forall x \geq 0, \) let \( P_x \) be the horizontal plane that is \( x \) units above \( P \). Suppose that \( S \) lies between \( P_1 \) and \( P_2 \). Suppose, also, that \( \forall x \in [1, 2], \) the intersection of \( S \) and \( P_x \) is the region inside an ellipse

whose major axis has length \( x \)
and whose minor axis has length \( e^{2x^2} \).

**Compute** the volume of \( S \).

**Hint:** Remember that if \( a \) and \( b \) are the major and minor axes of an ellipse \( E \), then the area inside \( E \) is \( \pi ab/4 \).

**ANSWER:**

Area of the ellipse in \( P_x \): \( \pi x e^{2x^2} / 4 \)

Volume of \( S \): \( \int_1^2 \left( \pi x e^{2x^2} / 4 \right) \, dx \)
Compute the volume of $S$.

**ANSWER:** Volume of $S$: \[
\int_1^2 \left( \frac{\pi x e^{2x^2}}{4} \right) \, dx
\]

Let $u = x^2$. Then $du = 2x \, dx$.

\[
\int_1^2 \left( \frac{\pi x e^{2x^2}}{4} \right) \, dx = \int_{1^2}^{2^2} \left( \frac{\pi e^{2u}}{4} \right) \, \frac{du}{2}
\]

\[
= \frac{\pi}{8} \left[ e^{2u} \right]_{u:1}^{4}
\]

\[
= \frac{\pi}{8} \left[ \frac{e^8}{2} - \frac{e^2}{2} \right]
\]
Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.
0720-12. Using the disk method, find the volume in a ball of radius 5, following the diagram shown below.

\[ y = \sqrt{5^2 - x^2} \]

**ANSWER:** \[ \int_{-5}^{5} \pi \left( \sqrt{5^2 - x^2} \right)^2 \, dx \]
Using the disk method, find the volume in a ball of radius 5.

**ANSWER:**

\[
\int_{-5}^{5} \pi \left( \sqrt{25 - x^2} \right)^2 \, dx = \pi \int_{-5}^{5} 5^2 - x^2 \, dx
\]

\[
= 2\pi \int_{0}^{5} 5^2 - x^2 \, dx
\]

\[
= 2\pi \left[ 5^2 x - \frac{x^3}{3} \right]_{x: \to 5}^{x: \to 0}
\]

\[
= 2\pi \left[ \left( 5^2 \cdot 5 - \frac{5^3}{3} \right) - (0) \right]
\]

\[
= 2\pi \left[ 5^3 - \frac{5^3}{3} \right]
\]
Using the disk method, find the volume in a ball of radius 5, ... 

**ANSWER:**

\[
\int_{-5}^{5} \pi \left( \sqrt{25 - x^2} \right)^2 \, dx = 2\pi \left[ 5^3 - \frac{5^3}{3} \right]
\]

\[
= 2\pi \left[ \frac{1}{3} \right] \left[ 5^3 \right]
\]

\[
= \frac{4}{3}\pi \left[ 5^3 \right]
\]
We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.
We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the washer method, find its volume.

\[ y = \sqrt{5^2 - x^2} \]

ANSWER: \[ \int_{-3}^{3} \pi \left( \sqrt{5^2 - x^2} \right)^2 - \pi [4^2] \, dx \]
We create a napkin holder . . .

Using the washer method, find its volume.

\[
\text{ANSWER: } \int_{-3}^{3} \pi \left( \sqrt{5^2 - x^2} \right)^2 - \pi \left[ 4^2 \right] \, dx
\]

\[
= \pi \int_{-3}^{3} 5^2 - x^2 - 4^2 \, dx
\]

\[
= \pi \int_{-3}^{3} 3^2 - x^2 \, dx
\]

\[
= 2\pi \int_{0}^{3} 3^2 - x^2 \, dx
\]

\[
= 2\pi \left[ 3^2 x - \frac{x^3}{3} \right]_{x: \rightarrow 3}
\]

\[
= 2\pi \left[ \left( 3^2 \cdot 3 - \frac{3^3}{3} \right) - (0) \right]
\]
Using the washer method, find its volume.

\[
\text{ANSWER: } \int_{-3}^{3} \pi \left( \sqrt{5^2 - x^2} \right)^2 - \pi \left[ 4^2 \right] \, dx
\]

\[
= 2\pi \left[ \left( 3^2 \cdot 3 - \frac{3^3}{3} \right) - (0) \right]
\]

\[
= 2\pi \left[ 3^3 - \frac{3^3}{3} \right]
\]

\[
= 2\pi \left[ 1 - \frac{1}{3} \right] \left[ 3^3 \right]
\]

\[
= 2\pi \left[ \frac{2}{3} \right] \left[ 3^3 \right]
\]

\[
= \frac{4}{3} \pi \left[ 3^3 \right]
\]