CALCULUS
Volume by cylindrical shells:
Problems
OLD
Using the shell method, find the volume in a ball of radius 5, following the diagram shown below.
0750-1. Using the shell method, find the volume in a ball of radius 5, following the diagram shown below.

**Answer:**

\[ \int_{0}^{5} [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \, dy \]
0750-1. Using the shell method, find the volume in a ball of radius 5, following the diagram . . .

**ANSWER:** \(z = 5^2 - y^2, \quad dz = -2y\,dy\)

\[
\int_0^5 [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \,dy = -2\pi \int_{5^2}^0 \sqrt{z} \,dz
\]

\[
= 2\pi \int_0^{5^2} \sqrt{z} \,dz
\]

\[
= 2\pi \int_0^{5^2} z^{1/2} \,dz
\]

\[
= 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \to 5^2}^{z: \to 0}
\]
Using the shell method, find the volume in a ball of radius 5, following the diagram...

**Answer:**

\[ z = 5^2 - y^2, \quad dz = -2y \, dy \]

\[
\int_0^5 [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \, dy = 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \to 5^2}^{z: \to 0}
\]

\[
= 2\pi \left[ \frac{(5^2)^{3/2}}{3/2} - 0 \right]
\]

\[
= \frac{4}{3} \pi \left[ 5^3 \right] \]

\[
= \frac{4}{3} \pi \left[ 5^3 \right] \]

\[ 5 \]
0750-2. We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the shell method, find its volume.
We create a napkin holder by drilling a cylindrical hole of radius 4 through the middle of a ball of radius 5, as shown below. Using the shell method, find its volume.

\[
y = \sqrt{5^2 - x^2}
\]

\[
\sqrt{5^2 - y^2}
\]

\[
\int_{4}^{5} [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \, dy
\]

**ANSWER:** \[
\int_{4}^{5} [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \, dy
\]
We create a napkin holder... Using the shell method, find its volume.

**ANSWER:** \( z = 5^2 - y^2, \quad dz = -2y \, dy \)

\[
\int_4^5 [2\pi y] \left[ 2\sqrt{5^2 - y^2} \right] \, dy = -2\pi \int_{5^2-4^2}^0 \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{5^2-4^2} \sqrt{z} \, dz
\]

\[
= 2\pi \int_0^{3^2} z^{1/2} \, dz
\]

\[
= 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z:0}^{3^2}
\]
We create a napkin holder . . .

Using the shell method, find its volume.

**ANSWER:** \( z = 5^2 - y^2 \), \( \;\;\; dz = -2y \, dy \)

\[
\int_{4}^{5} \left[ 2\pi y \right] \left[ 2\sqrt{5^2 - y^2} \right] \, dy = 2\pi \left[ \frac{z^{3/2}}{3/2} \right]_{z: \rightarrow 0}^{z: \rightarrow 3^2}
\]

\[
= 2\pi \left[ \frac{(3^2)^{3/2}}{3/2} \right]
\]

\[
= \frac{4}{3} \pi \left[ 3^3 \right]
\]
Let \( R \) be the region bounded by 
\[ y = (x - 1)^2(x - 2)^2 \text{ and } y = 4. \]

a. Sketch \( R \).

b. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the \( x \)-axis. \textbf{Do not evaluate} the integral.

c. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the \( y \)-axis. \textbf{Do not evaluate} the integral.

d. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating \( R \) about the line \( x = -1 \). \textbf{Do not evaluate} the integral.
Let \( R \) be the region bounded by 
\[ y = (x - 1)^2(x - 2)^2 \] 
and \( y = 4 \).

a. Sketch \( R \).
Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the $x$-axis. Do not evaluate the integral.

$$b. \int_{0}^{3} \pi \left( 4^2 \right) - \pi \left( (x - 1)^2(x - 2)^2 \right)^2 \, dx$$
0750-3. c. Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the $y$-axis. Do not evaluate the integral.

$$\int_0^3 [2\pi x] \left[ 4 - (x - 1)^2(x - 2)^2 \right] \, dx$$

$y = 4$

$y = (x - 1)^2(x - 2)^2$
Using whatever method you prefer, set up an integral to compute the volume of the solid obtained by rotating $R$ about the line $x = -1$. Do not evaluate the integral.

$$\int_0^3 [2\pi(x + 1)][4 - (x - 1)^2(x - 2)^2] \, dx$$
0750-4. Let $R$ be the region bounded by 
\[ x = 1 + e^{-y^2}, \quad x = 0, \quad y = 1 \text{ and } y = 2. \]

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**ANSWER:**

a. [Diagram showing the region $R$ defined by $x = 1 + e^{-y^2}$, $x = 0$, $y = 1$, and $y = 2$.]
Let $R$ be the region bounded by

$$x = 1 + e^{-y^2}, \ x = 0, \ y = 1 \text{ and } y = 2.$$ 

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**ANSWER:**

a. 

b. 

$$\int_1^2 [2\pi y] \left[1 + e^{-y^2}\right] \, dy$$

$$= 2\pi \left[ \int_1^2 y \, dy \right] + 2\pi \left[ \int_1^2 ye^{-y^2} \, dy \right]$$

$$= 2\pi \left[ \frac{y^2}{2} \right]_{y=1}^{y=2} + 2\pi \left[ \int_{-1}^{-4} e^u \frac{du}{-2} \right]$$

$$u = -y^2$$

$$du = -2y \, dy$$
0750-4. Let $R$ be the region bounded by 

$$x = 1 + e^{-y^2}, \ x = 0, \ y = 1 \ \text{and} \ y = 2.$$ 

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the $x$-axis.

**ANSWER:**

b. $\int_{1}^{2} \left[ 2\pi y \right] \left[ 1 + e^{-y^2} \right] \, dy$

$$= 2\pi \left[ \frac{y^2}{2} \right]_{y:1}^{y:2} + 2\pi \left[ \int_{-1}^{-4} e^u \, du \right]$$

$$= \pi \left[ 2^2 - 1^2 \right] - \pi \left[ \int_{-1}^{-4} e^u \, du \right]$$

$$= 3\pi - \pi \left[ e^u \right]_{u:-4}^{u:-1}$$
Let \( R \) be the region bounded by 
\[ x = 1 + e^{-y^2}, \ x = 0, \ y = 1 \ \text{and} \ y = 2. \]

a. Sketch \( R \).

b. Using whatever method you prefer, find the volume of the solid obtained by rotating \( R \) about the \( x \)-axis.

\[
\text{Answer:}
\]

\[
\text{b. } \int_1^2 [2\pi y] \left[ 1 + e^{-y^2} \right] \, dy
\]

\[
= 3\pi - \pi \left[ e^u \right]_{u : -4}^{u : -1}
\]

\[
= 3\pi - \pi \left[ e^{-4} - e^{-1} \right]
\]

\[
= \pi \left[ 3 - e^{-4} + e^{-1} \right]
\]
Let $R$ be the region bounded by $x = y^2 + y$, $x = 0$, $y = 1$ and $y = 2$.

a. Sketch $R$.

b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$. 
Let \( R \) be the region bounded by \( x = y^2 + y, \ x = 0, \ y = 1 \) and \( y = 2 \).

a. Sketch \( R \).

ANS: a.
b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

\[ x = -1 \quad y \quad x = 0 \quad R \]  
\[ \text{WASHER} \]

\[ y = 2 \quad y = 1 \]

\[ x = y^2 + y \]

**ANSWER:**
\[ b. \int_1^2 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy \]
\[ = \pi \int_1^2 \left[ 1 + y^4 + y^2 + 2y^2 + 2y + 2y^3 \right] - 1 \, dy \]
b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

**ANSWER:**

\[
\int_{1}^{2} \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy
\]

\[
= \pi \int_{1}^{2} \left[ 1 + y^4 + y^2 + 2y^2 + 2y + 2y^3 \right] - 1 \, dy
\]

\[
= \pi \int_{1}^{2} y^4 + 2y^3 + 3y^2 + 2y \, dy
\]

\[
= \pi \left[ \frac{y^5}{5} + \frac{y^4}{2} + y^3 + y^2 \right]_{y=1}^{y=2}
\]
b. Using whatever method you prefer, find the volume of the solid obtained by rotating \( R \) about the line \( x = -1 \).

\[
\text{ANSWER: } \int_1^2 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy
\]

\[
= \pi \left[ \frac{y^5}{5} + \frac{y^4}{2} + y^3 + y^2 \right]_{y: \rightarrow 2}^{y: \rightarrow 1}
\]

\[
= \pi \left[ \frac{2^5 - 1^5}{5} + \frac{2^4 - 1^4}{2} + 2^3 - 1^3 + 2^2 - 1^2 \right]
\]

\[
= \pi \left[ \frac{31}{5} + \frac{15}{2} + 7 + 3 \right]
\]
b. Using whatever method you prefer, find the volume of the solid obtained by rotating $R$ about the line $x = -1$.

**ANSWER:**

\[ \int_1^2 \pi \left[ 1 + y^2 + y \right]^2 - \pi \left[ 1^2 \right] \, dy \]

\[ = \pi \left[ \frac{31}{5} + \frac{15}{2} + 7 + 3 \right] \]

\[ = \pi \left[ \frac{31}{5} + \frac{15}{2} + 10 \right] \]

\[ = \pi \left[ \frac{62}{10} + \frac{75}{10} + \frac{100}{10} \right] \]

\[ = \frac{237\pi}{10} \]
0750-6. Let \( R \) be the region bounded by 
\[ x = \sin y, \ x = 0, \ y = \pi/4 \text{ and } y = 3\pi/4. \]
Set up, but do not evaluate, an integral that yields the volume of the solid obtained by rotating \( R \) about the line \( y = \pi \).

**ANS:**

\[
\text{VOLUME} = \int_{\pi/4}^{3\pi/4} [2\pi(\pi - y)] [\sin y] \, dy
\]
0750-7. Describe the solid of revolution whose volume is given by

\[ 2\pi \int_1^2 x \left[ (e^{5x}) - (\cos(\pi x)) \right] \, dx. \]

Do not evaluate this integral.

**ANSWER:**

This is the solid of revolution obtained by revolving, about the $y$-axis, the region bounded by

\[ y = e^{5x}, \quad y = \cos(\pi x), \quad x = 1 \text{ and } x = 2. \]

**NOTE:** This is the most natural answer, given the problem, but other answers are correct.
0750-8. Describe the solid of revolution whose volume is given by

\[ 2\pi \int_1^2 [x + 3] \left[ (e^{5x}) - (\cos(\pi x)) \right] \, dx. \]

**Do not evaluate this integral.**

**ANSWER:**
This is the solid of revolution obtained by revolving, about the line \( x = -3 \), the region bounded by

\[ y = e^{5x}, \quad y = \cos(\pi x), \quad x = 1 \text{ and } x = 2. \]

**NOTE:** This is the most natural answer, given the problem, but other answers are correct.