1. (40 points) Consider the function

\[ f(x) = \frac{1}{(2 + x)^2}. \]

By using the definition of the derivative, find \( f'(2) \). You are not allowed to use the differentiation rules.

\[
\frac{f(2+h) - f(2)}{h} = \frac{1}{h} \left[ \frac{1}{(4+h)^2} - \frac{1}{4^2} \right] = \frac{1}{h} \left[ \frac{4^2 - (4+h)^2}{(4+h)^2 \cdot 4^2} \right]
\]

\[
= \frac{1}{h} \left[ \frac{4^2 - (4^2 + 8h + h^2)}{(4+h)^2 \cdot 4^2} \right] = \frac{1}{h} \left[ \frac{-8h - h^2}{(4+h)^2 \cdot 4^2} \right]
\]

\[
= \frac{-8 - h}{(4+h)^2 \cdot 4^2} \quad \text{as} \quad h \to 0 \quad \Rightarrow \quad \frac{-8}{4^2 \cdot 4^2} = -\frac{8}{16 \cdot 16} = -\frac{1}{2 \cdot 16} = -\frac{1}{32}
\]

2. (15 points) Is the following statement true or false?

\[ \lim_{x \to -\infty} \ln \left( \frac{-1}{x} \right) = -\infty \]

\[ \text{True} \]

\[ \text{False} \]

\[ \left\{ \begin{array}{c}
\frac{-1}{-\infty} = 0^+ \text{) } \quad \text{and} \\
\ln (0^+) = -\infty \text{) }
\end{array} \right. \]

SEE OTHER SIDE FOR MORE PROBLEMS.
3. (25 points) Find
\[
\lim_{x \to \infty} \frac{4x^2 + 7x - 8x^3 + 19}{3x^2 + 6x + 5}.
\]

**Asymptotics**
\[
\lim_{x \to \infty} \frac{-8x^3}{3x^2} = \lim_{x \to \infty} -\frac{8}{3}x = -\infty
\]

\[
\left(\frac{-8}{3}\cdot\infty\right) = -\infty
\]

4. (20 points) Suppose \(f(x)\) has the following graph.

Which of the following is true about the derivative function \(f'(x)\)?

(A) \(f'(-2) > 0. \) \( \times \)

(B) \(f'(-1) < 0. \) \( \checkmark \)

(C) \(f'(3) < 0. \) \( \times \)

(D) \(f'(5) = 0. \) \( \times \)

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