

CALCULUS

Simple limit problems

Exercise 1: Use the graph of f given below to find the value of each quantity, if it exists. If it does not exist, explain why.

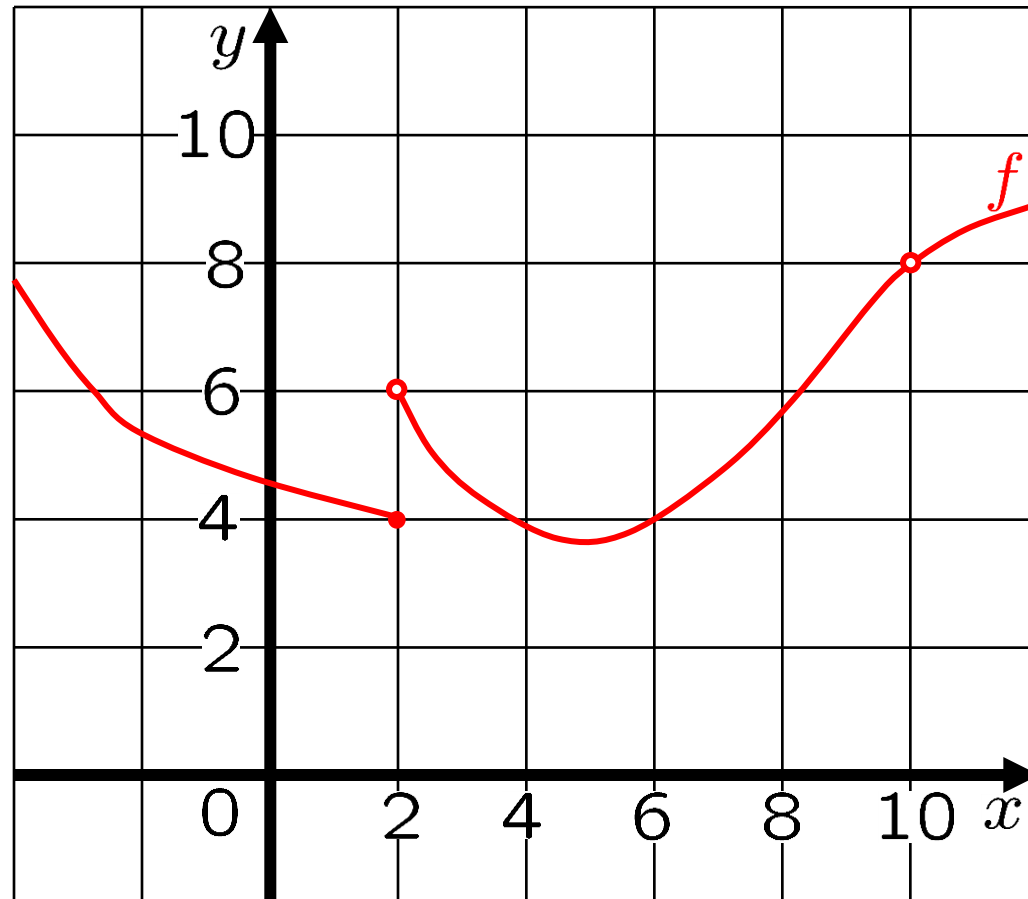
(a) $\lim_{x \rightarrow 2^-} f(x)$

(b) $\lim_{x \rightarrow 2^+} f(x)$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $\lim_{x \rightarrow 10} f(x)$

(e) $f(10)$



SKILL
lim from gph

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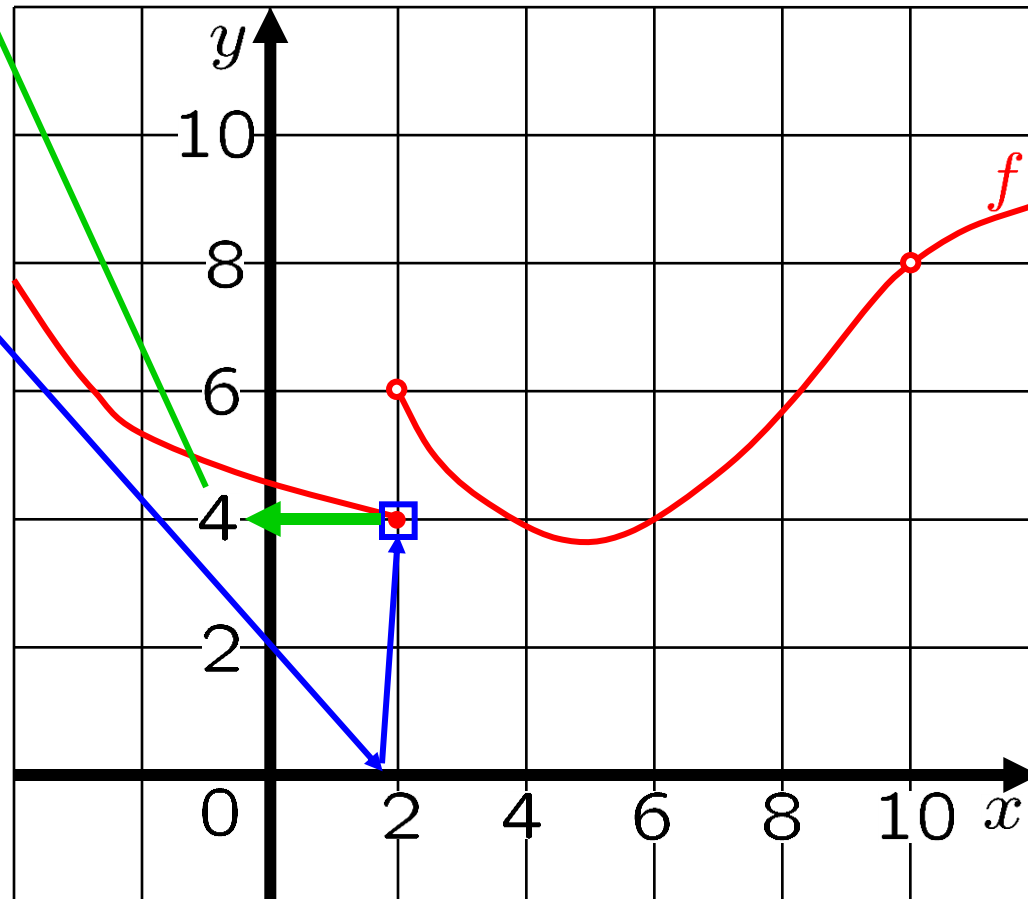
(a) $\lim_{x \rightarrow 2^-} f(x) = 4$

(b) $\lim_{x \rightarrow 2^+} f(x)$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $\lim_{x \rightarrow 10} f(x)$

(e) $f(10)$



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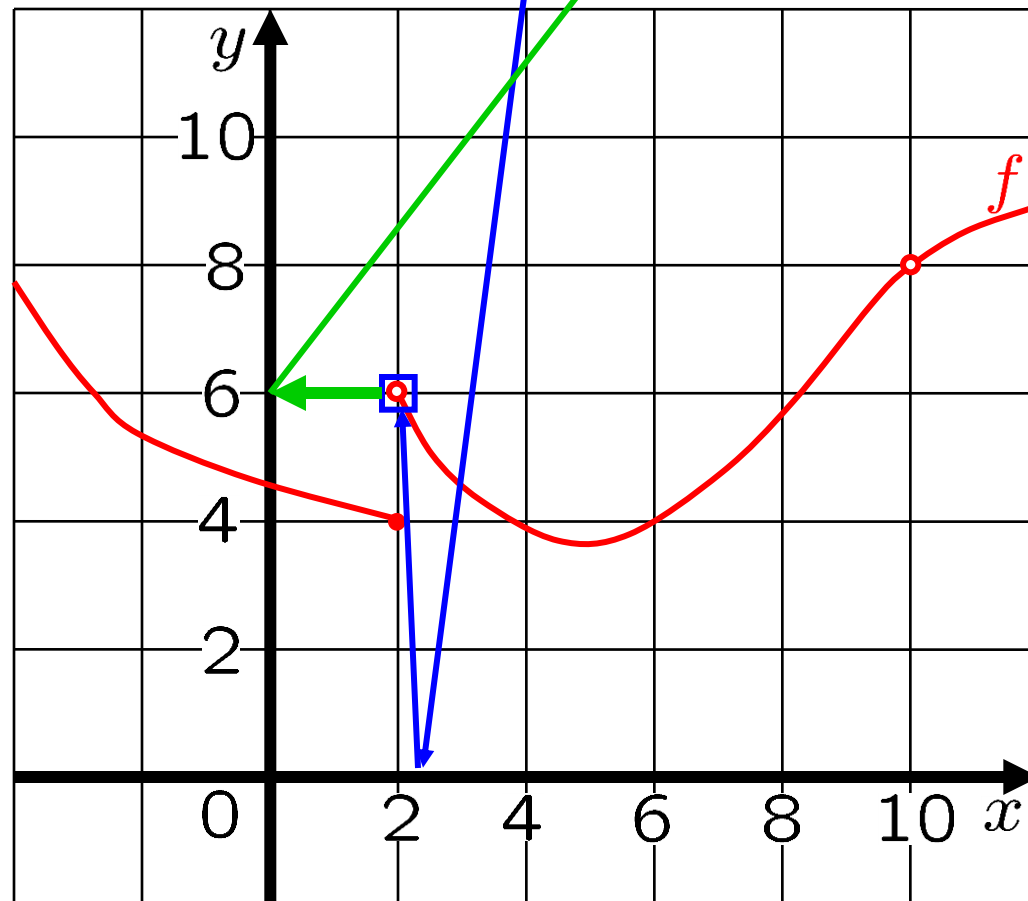
(a) $\lim_{x \rightarrow 2^-} f(x) = 4$

(b) $\lim_{x \rightarrow 2^+} f(x) = 6$

(c) $\lim_{x \rightarrow 2} f(x)$

(d) $\lim_{x \rightarrow 10} f(x)$

(e) $f(10)$



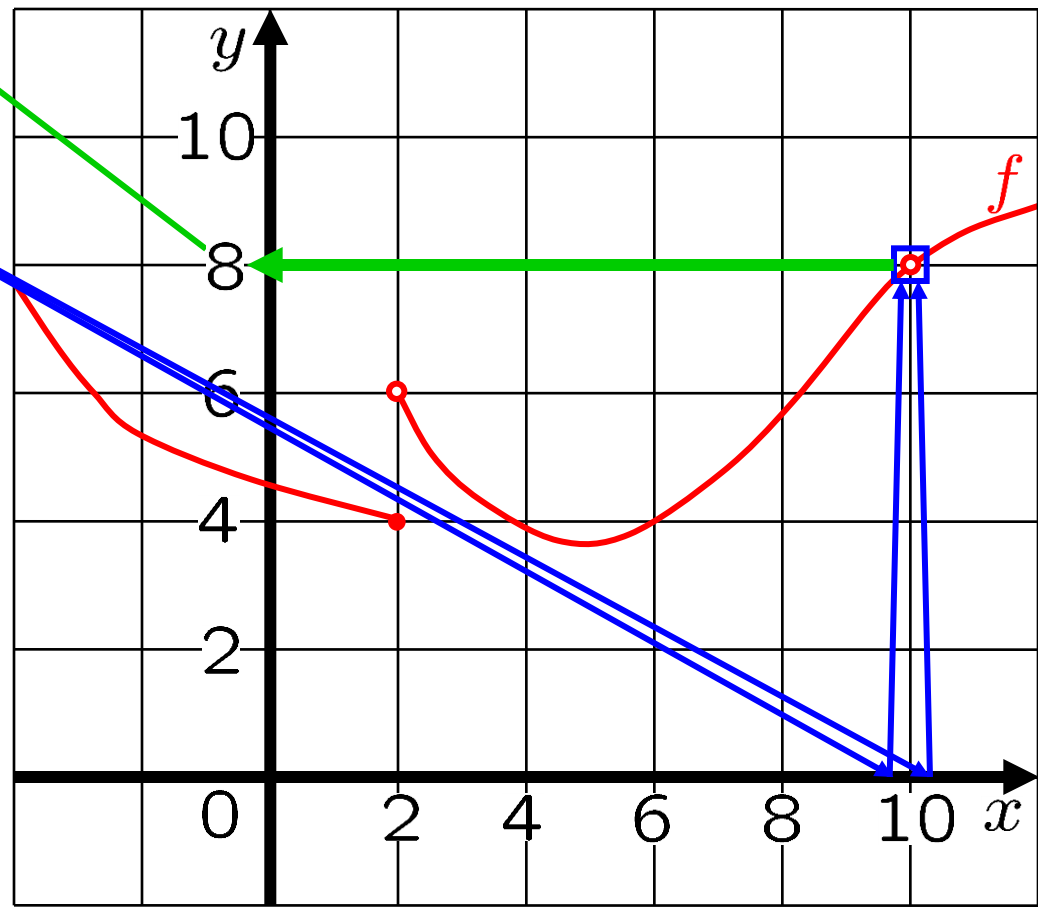
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- (a) $\lim_{x \rightarrow 2^-} f(x) = 4$ (b) $\lim_{x \rightarrow 2^+} f(x) = 6$ (c) $\lim_{x \rightarrow 2} f(x)$ does not exist

(d) $\lim_{x \rightarrow 10} f(x) = 8$

(e) $f(10)$



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Exercise 1: Use the graph of f given below to find the value of each quantity, if it exists.

If it does not exist, explain why.

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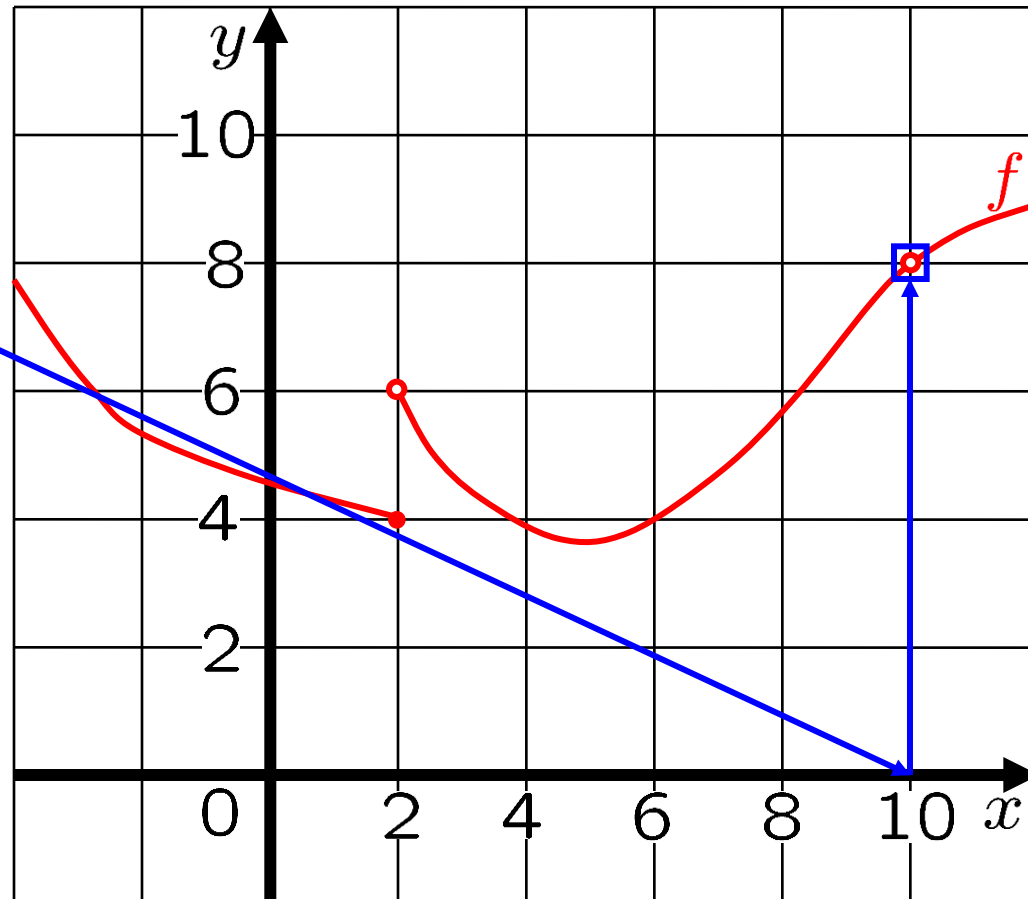
(b) $\lim_{x \rightarrow 2^+} f(x) = 6$

(c) $\lim_{x \rightarrow 2} f(x)$ does not exist

(d) $\lim_{x \rightarrow 10} f(x) = 8$

(e) $f(10)$ does not exist

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Exercise 2: Let H be the function whose graph is shown below. Find the following quantities:

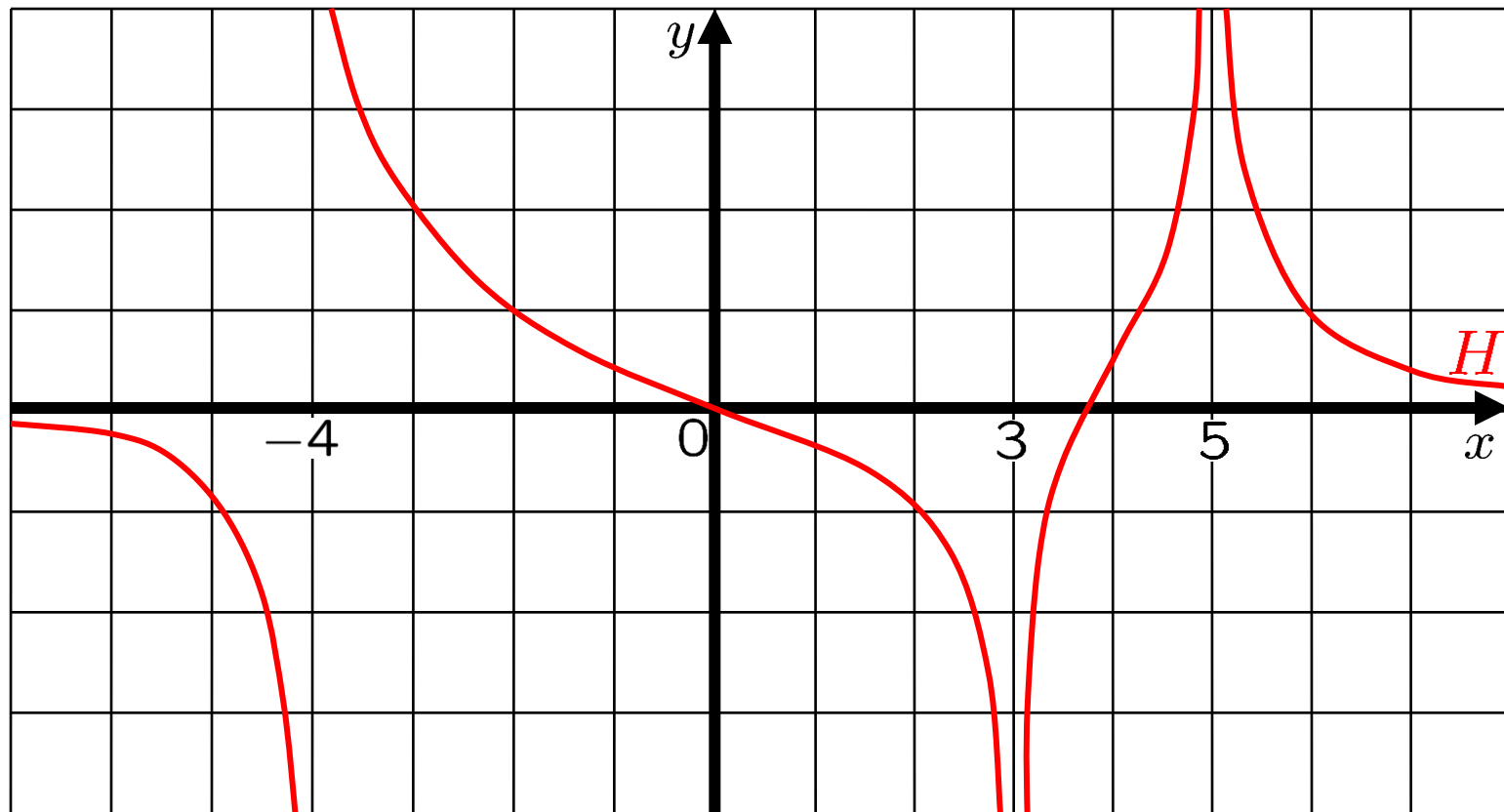
(a) $\lim_{x \rightarrow 3} H(x)$

(b) $\lim_{x \rightarrow 5} H(x)$

(c) $\lim_{x \rightarrow -4^-} H(x)$

(d) $\lim_{x \rightarrow -4^+} H(x)$

(e) The equations of the vertical asymptotes.



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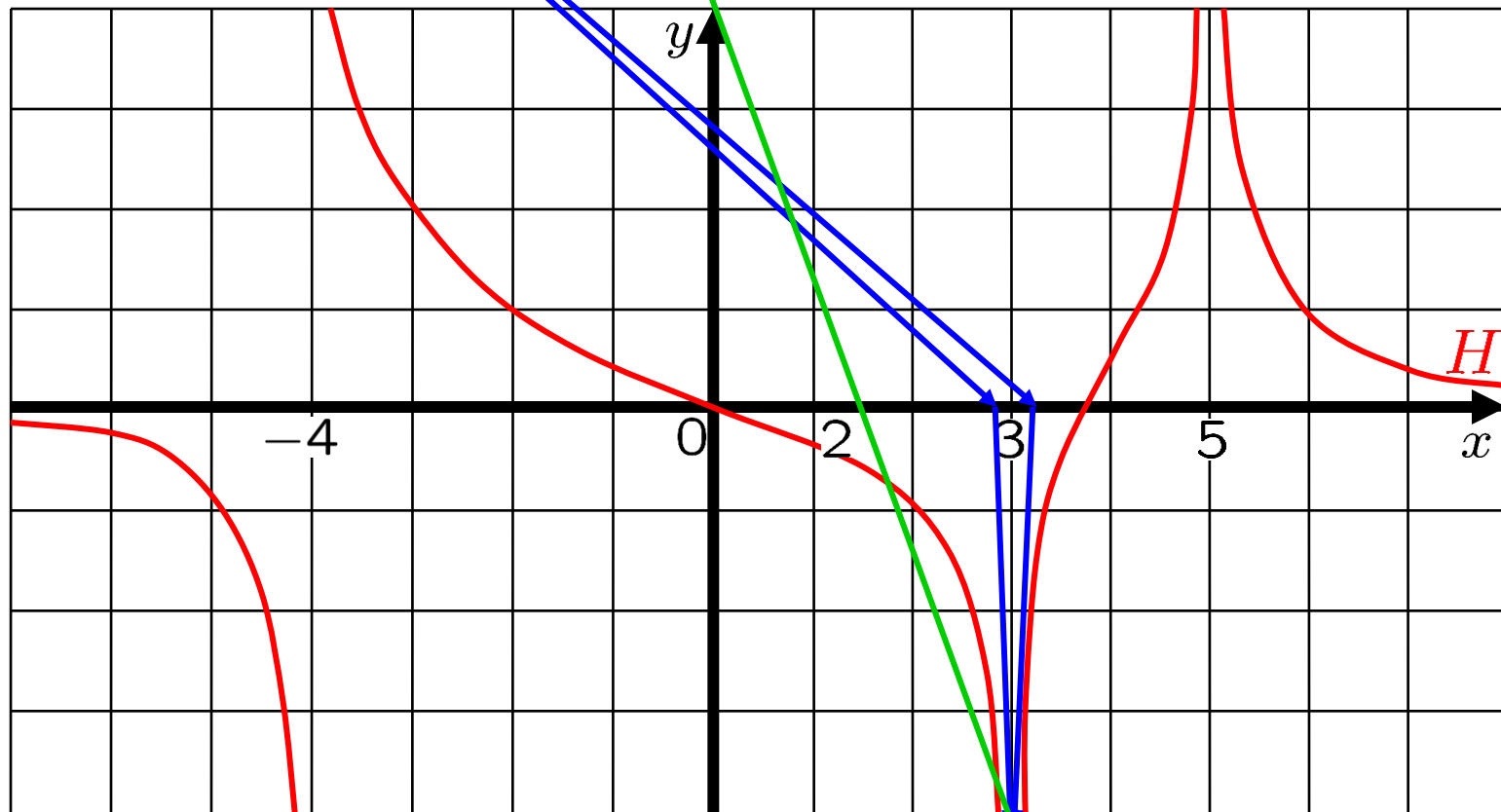
(a) $\lim_{x \rightarrow 3} H(x) = -\infty$

(b) $\lim_{x \rightarrow 5} H(x)$

(c) $\lim_{x \rightarrow -4^-} H(x)$

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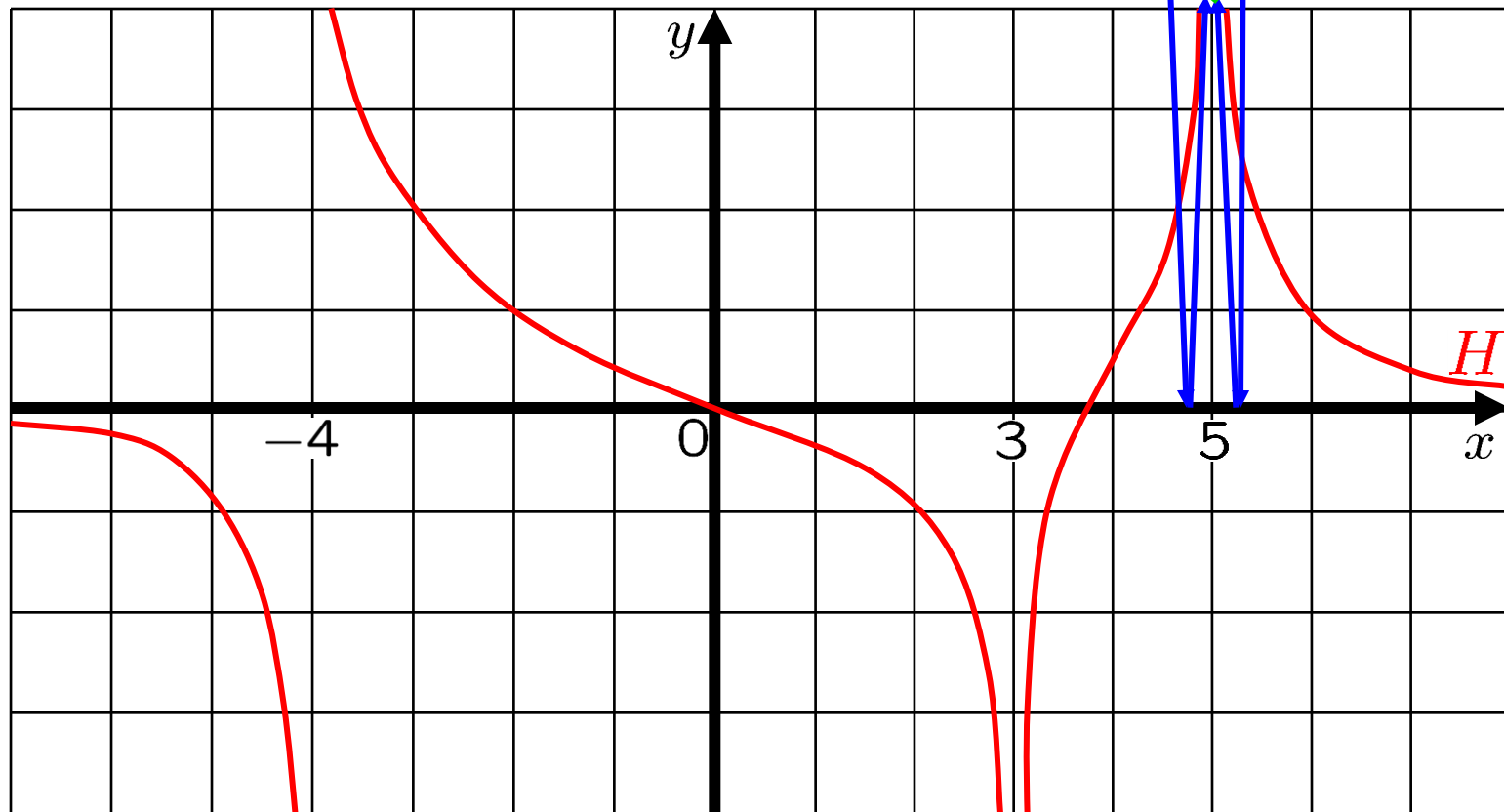
(a) $\lim_{x \rightarrow 3} H(x) = -\infty$

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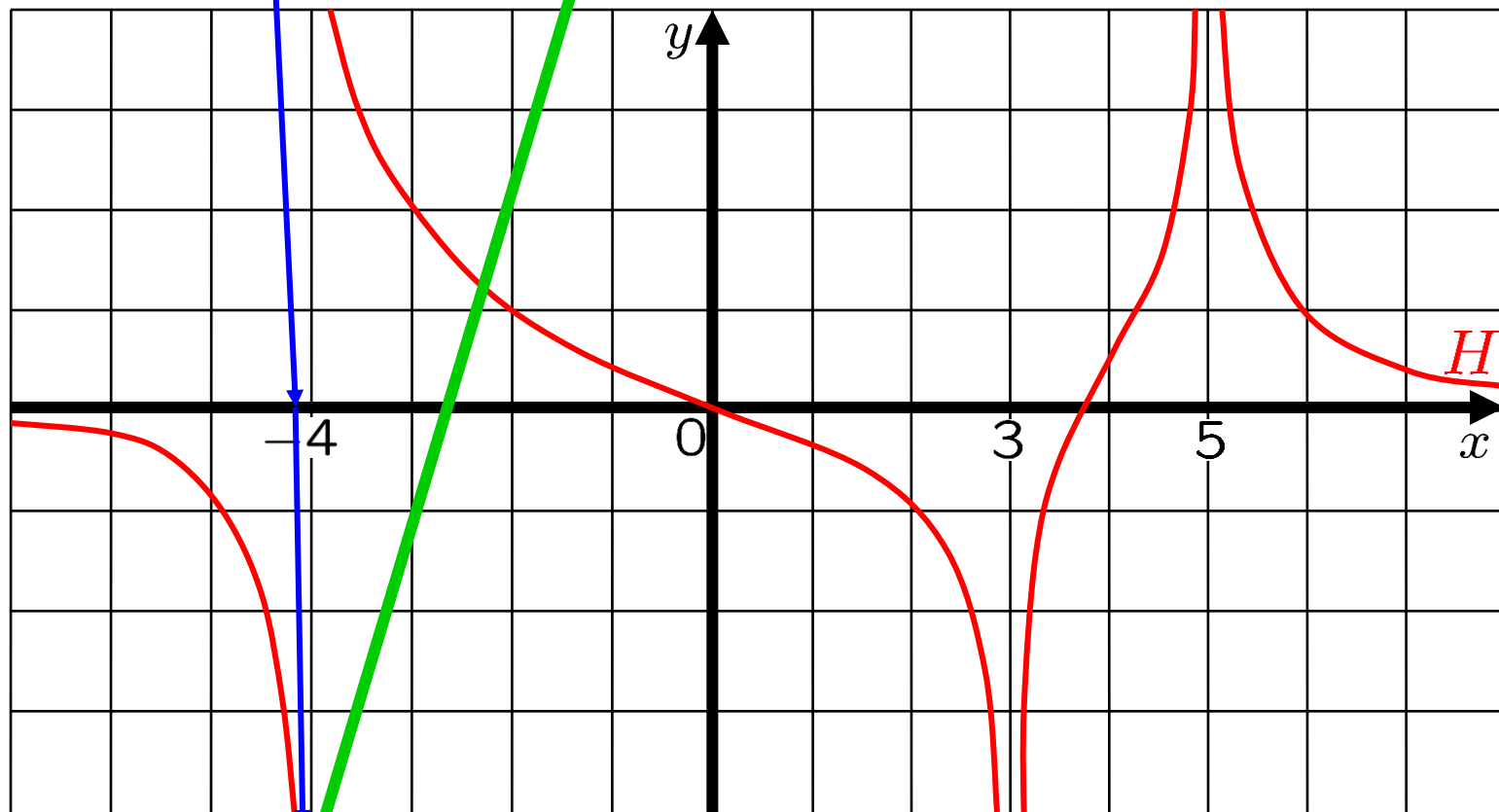
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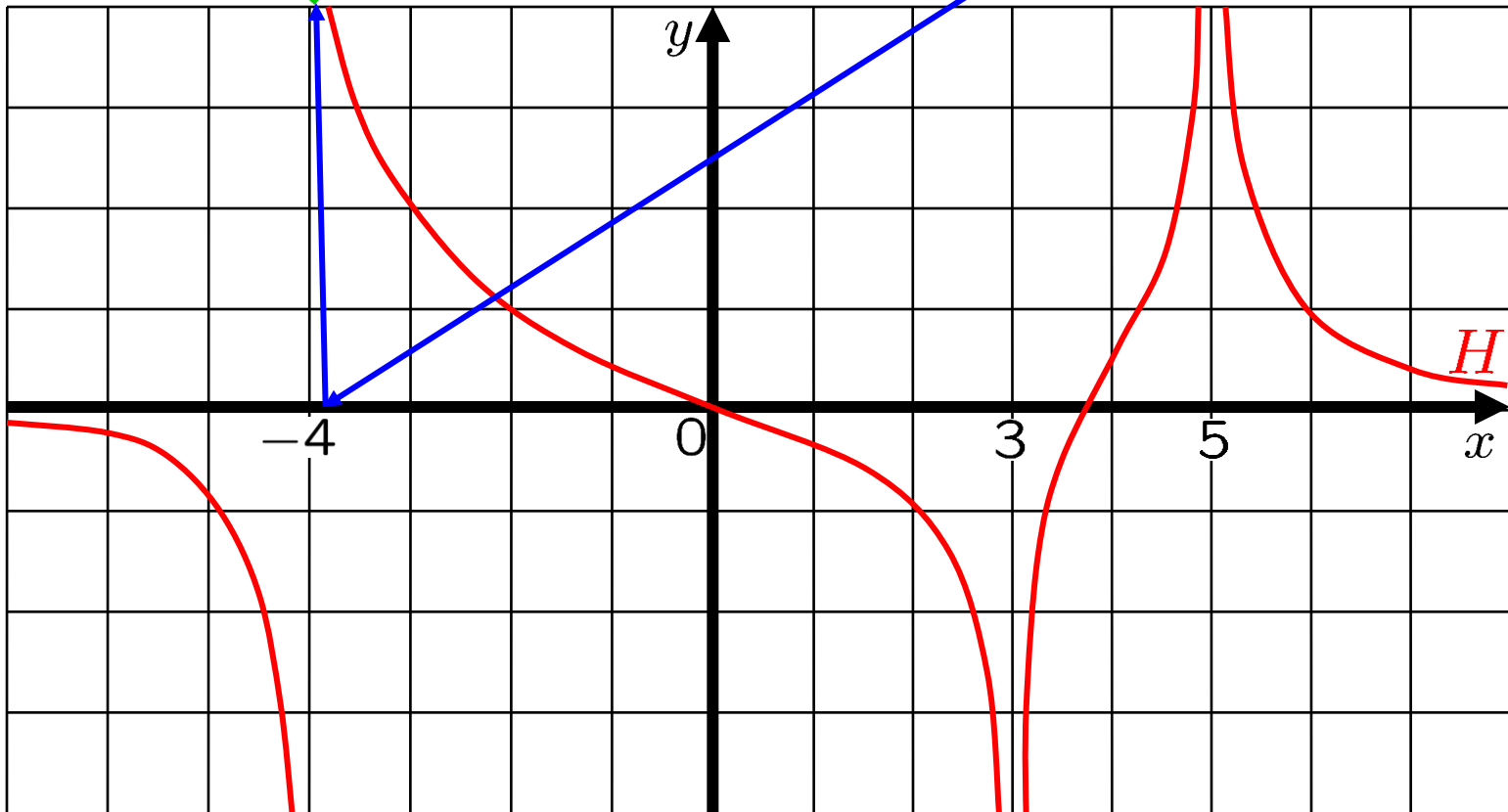
(e) The equations of the vertical asymptotes.



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lim from gph

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- (a) $\lim_{x \rightarrow 3} H(x) = -\infty$ (b) $\lim_{x \rightarrow 5} H(x) = \infty$
 $\lim_{x \rightarrow -4} R(x) = \text{DNE}$
(c) $\lim_{x \rightarrow -4^-} H(x) = -\infty$ (d) $\lim_{x \rightarrow -4^+} H(x) = \infty$
(e) The equations of the vertical asymptotes.



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(a) $\lim_{x \rightarrow 3} H(x) = -\infty$

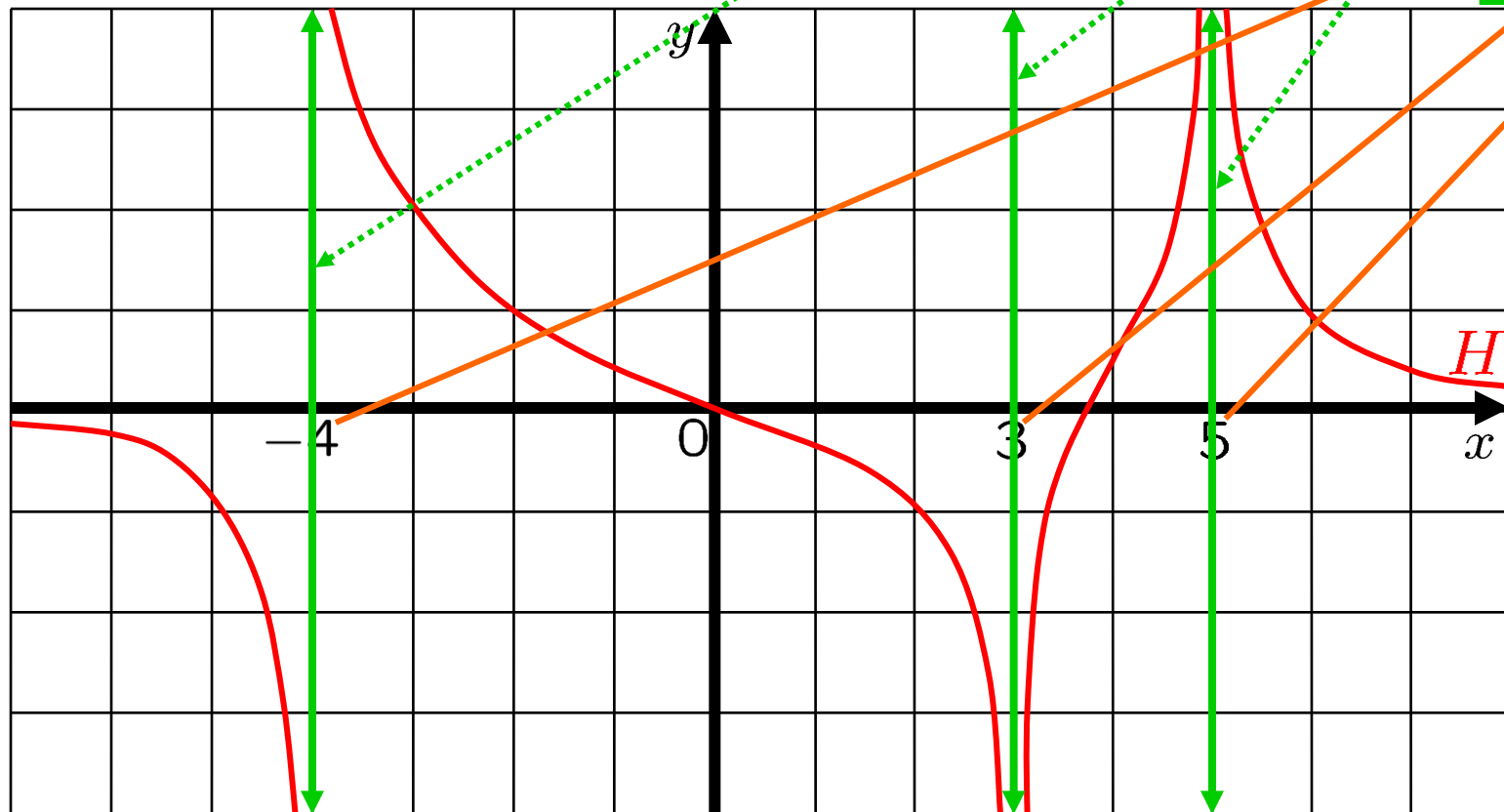
(b) $\lim_{x \rightarrow 5} H(x) = \infty$

(c) $\lim_{x \rightarrow -4^-} H(x) = -\infty$

(d) $\lim_{x \rightarrow -4^+} H(x) = \infty$

(e) The equations of the vertical asymptotes.

$x = -4$
 $x = 3$
 $x = 5$

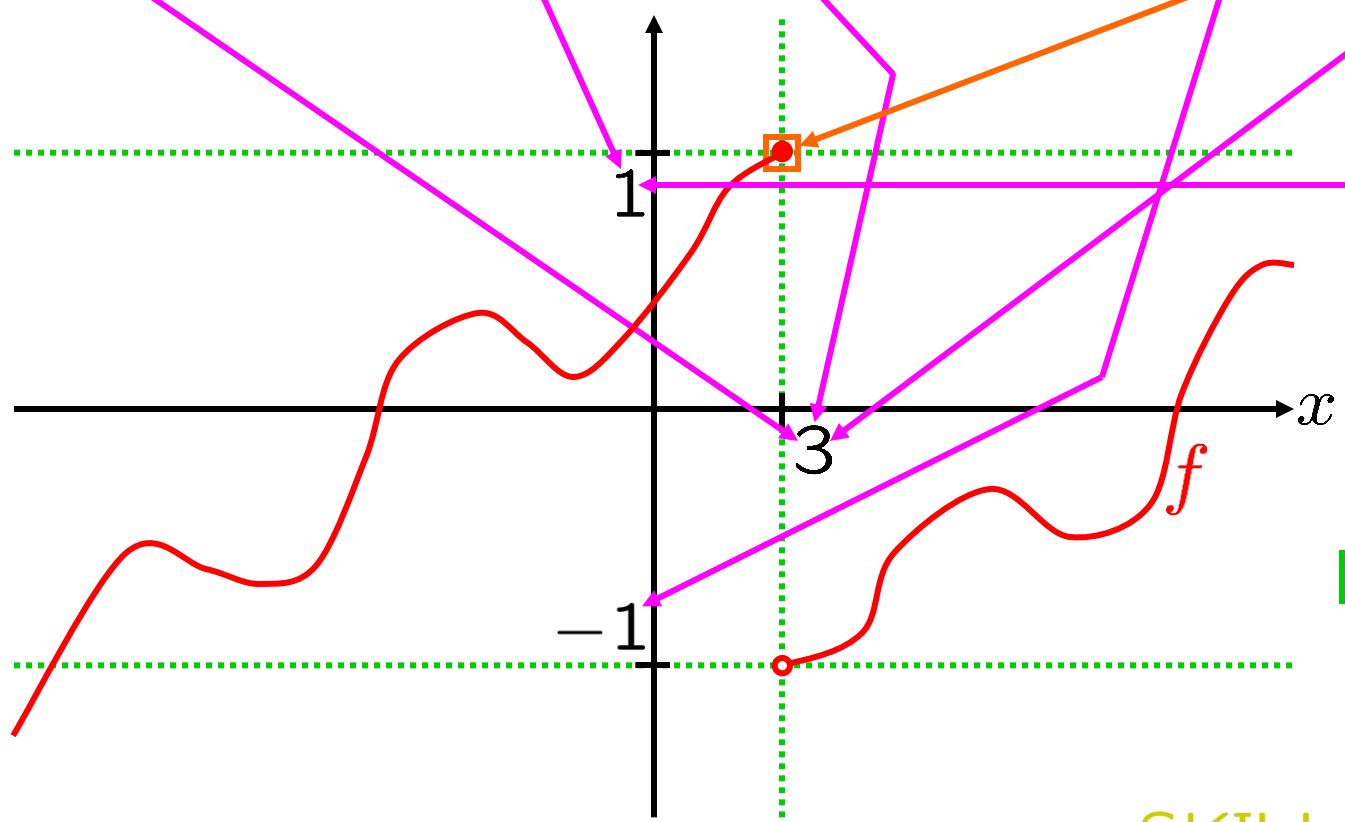


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Exercise 3:

Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x \rightarrow 3^-} f(x) = 1, \quad \lim_{x \rightarrow 3^+} f(x) = -1, \quad f(3) = 1$$



Many,
many
other
correct
answers!

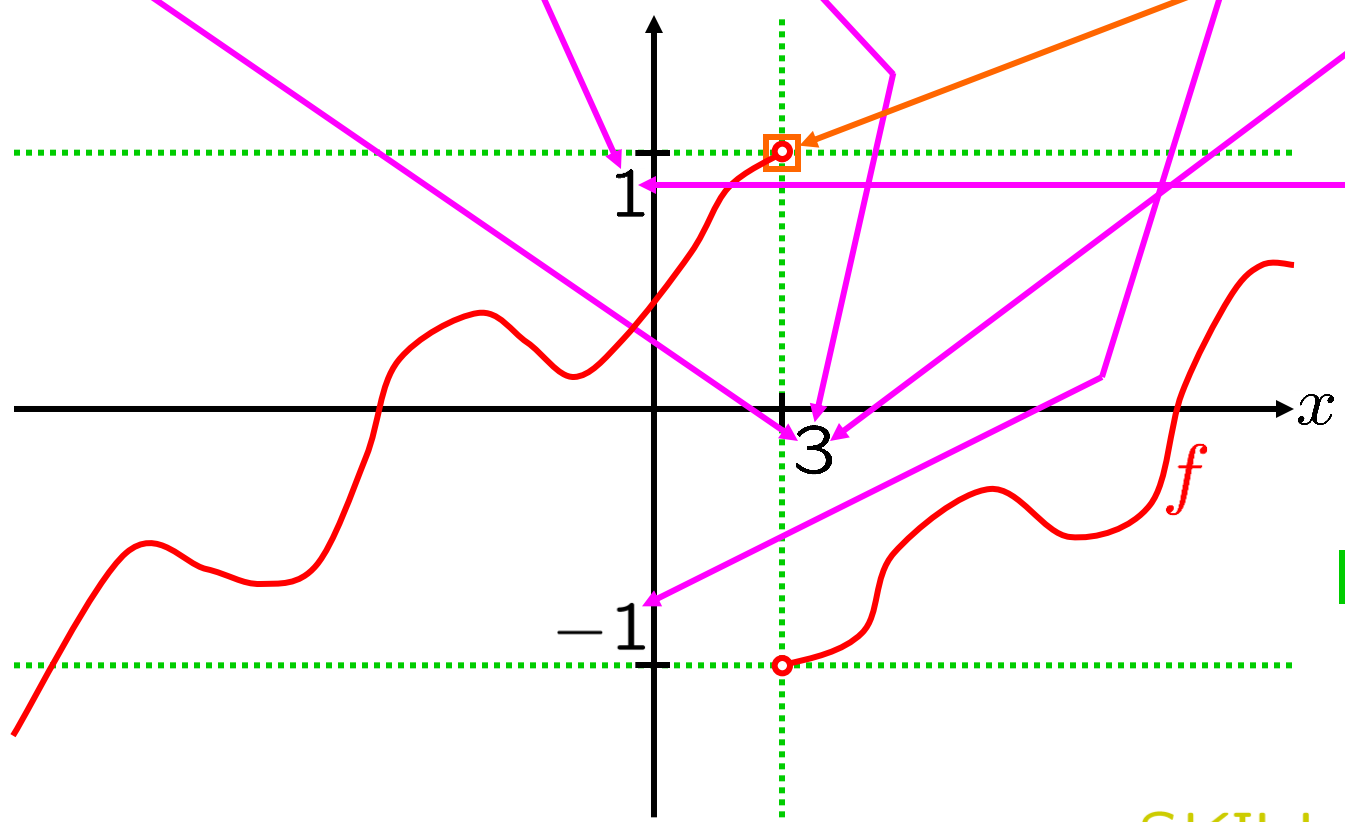


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Exercise 4:

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Determine the infinite limit.

one-sided limit

$$\lim_{x \rightarrow -5^-} \frac{x+3}{x+5} = \infty$$

quotient of polynomials
rational expression

A one-sided limit of a rational expression with
denom. $\rightarrow 0$ and num. $\neq 0$
is ∞ or $-\infty$. Which?

denominator

$$x + 5 \rightarrow 0$$

$$\left[\frac{x+3}{x+5} \right]_{x: \rightarrow -5-0.001} = \frac{-5 - 0.001 + 3}{-5 - 0.001 + 5}$$

$$= \frac{+2.001}{+0.001}$$

$$= 2,001 \text{ is very positive}$$

Exercise 4:

Determine the infinite limit.

$$\lim_{x \rightarrow -5^-} \frac{x + 3}{x + 5} = \infty \quad \blacksquare$$

A **one-sided** limit of a rational expression with
denom. $\rightarrow 0$ and num. $\nrightarrow 0$
is ∞ or $-\infty$. Which?

Another way to say the same thing:

$$\frac{x + 3}{x + 5}$$

negative
small negative

$x \rightarrow -5^-$

A one-sided limit of a rational expression with
denom. $\rightarrow 0$ and num. $\neq 0$
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$$\lim_{x \rightarrow -5} \frac{x + 3}{x + 5}$$

$\frac{\text{negative}}{\text{small positive}}$ $x \rightarrow -5^+ \rightarrow -\infty$
 $\frac{\text{negative}}{\text{small negative}}$ $x \rightarrow -5^- \rightarrow \infty$

does not exist.

A **one-sided** limit of a rational expression with
denom. $\rightarrow 0$ and num. $\neq 0$
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A **two-sided** limit of a rational expression with
denom. $\rightarrow 0$ and num. $\neq 0$
is ∞ or $-\infty$,
PROVIDED it exists.

$$\lim_{x \rightarrow -5} \frac{x + 3}{x + 5} \text{ does not exist.}$$

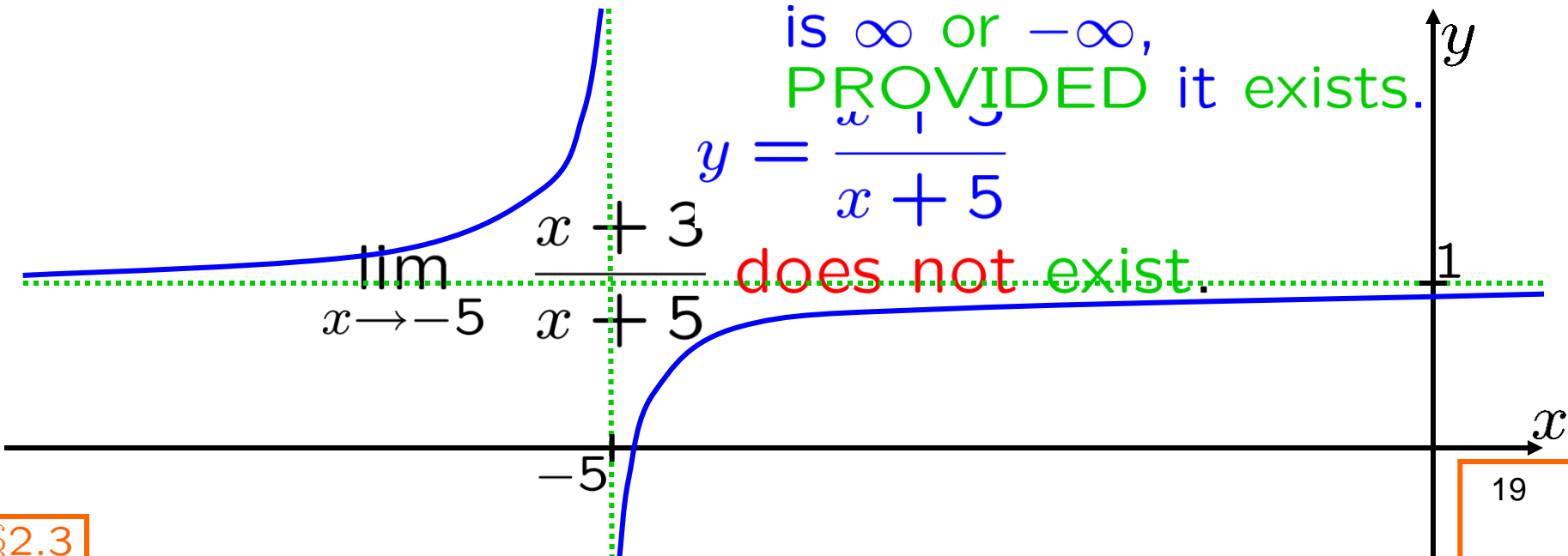
A **one-sided** limit of a rational expression with
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A **two-sided** limit of a rational expression with
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 is ∞ or $-\infty$,
 PROVIDED it exists.



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A **two-sided** limit of a rational expression with
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PROVIDED it exists.

$$\lim_{x \rightarrow -5} \frac{x + 3}{x + 5} \text{ does not exist.}$$

