CALCULUS
Definition of logarithm

Some more functions. . . $10^\bullet$ expr. of x: 10^x

exponential functions: $10^\bullet$, $12^\bullet$, $5^\bullet$, $2^\bullet$, *etc.*

exponential expressions of x: 10^x, 12^x, 5^x, 2^x, *etc.*

NOTE: 2^x and x^2 are very different.

transcendental in x — polynomial in x

If $b \neq 0$ and $b \neq 1$, then b^x is transcendental in x.

$0^x \overset{x \neq 0}{=} 0$ 0^0 DNE

Next subtopic:
Gphs of exp fns

$0^x = \frac{0}{x}$ booooring

rational in x

$1^x = 1$ booooring

polynomial in x

0^x is not transcendental in x.

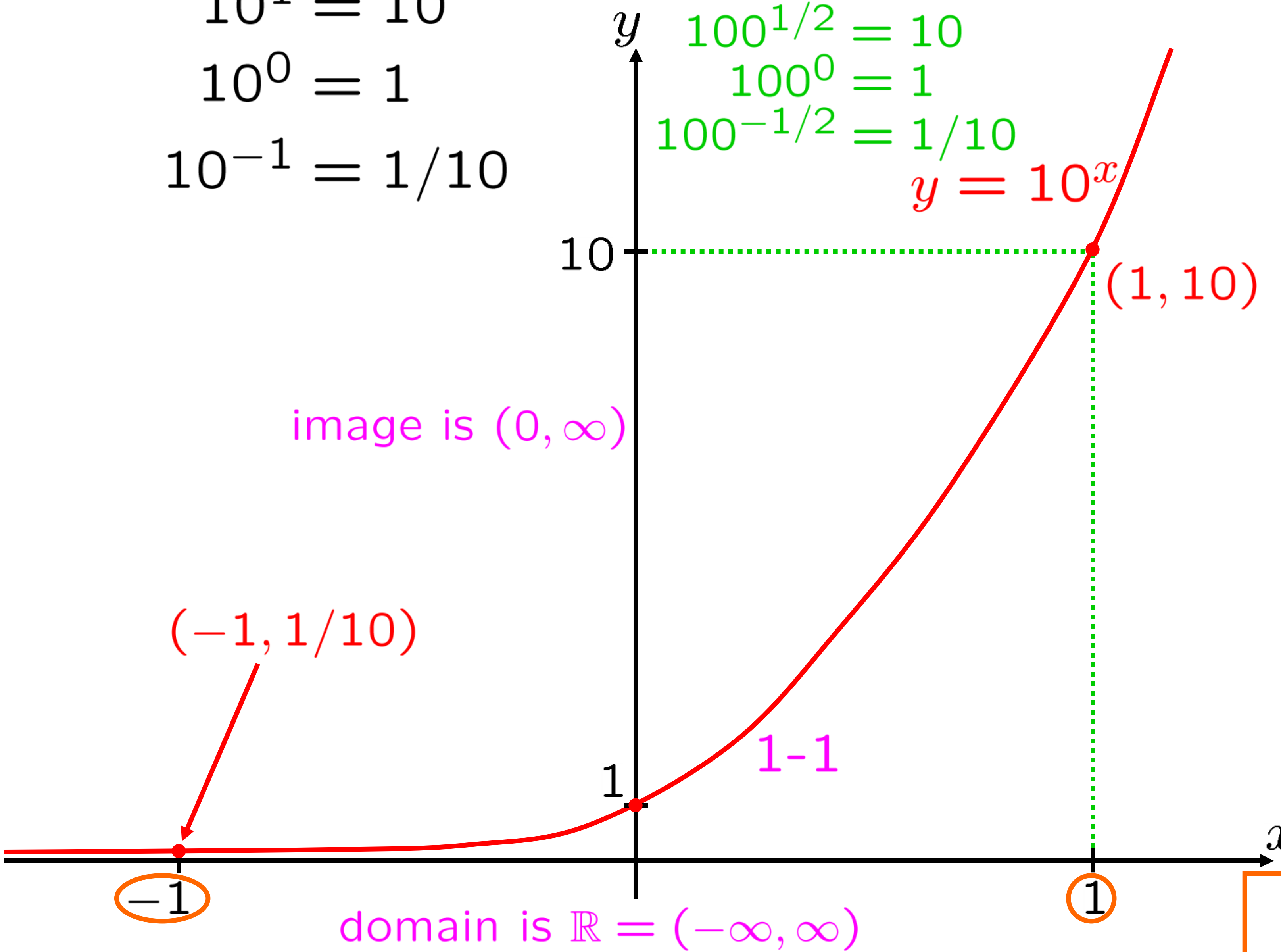
1^x is not transcendental in x.

Some more functions. . . $10^{\bullet}$ expr. of x: 10^x

Change to: $100^{\bullet}$

$$10^1 = 10$$
$$10^0 = 1$$
$$10^{-1} = 1/10$$

$$100^{1/2} = 10$$
$$100^0 = 1$$
$$100^{-1/2} = 1/10$$



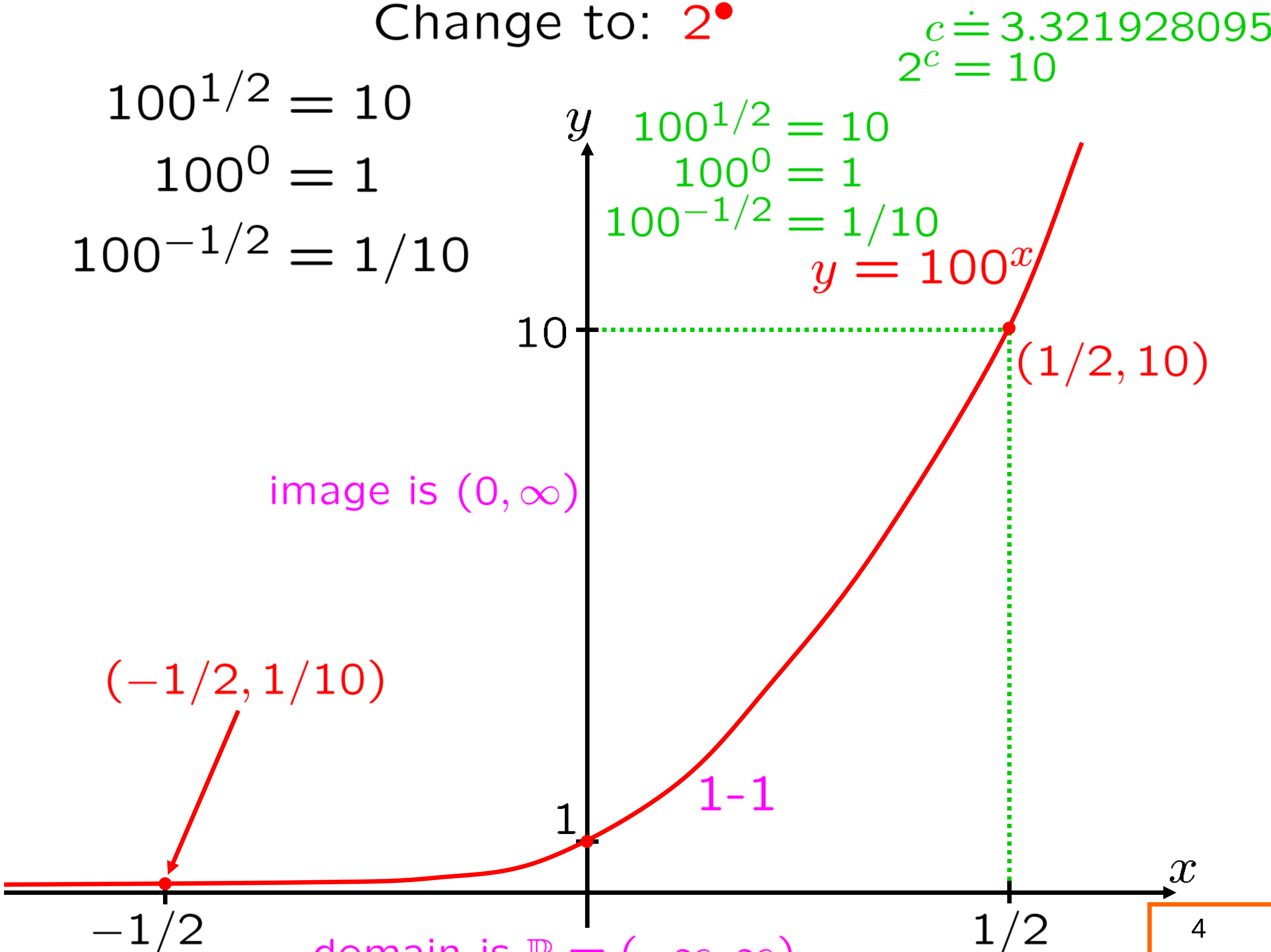
Some more functions... $100^{\bullet}$ expr. of x: 100^x

Change to: $2^{\bullet}$

$100^{1/2} = 10$

$100^0 = 1$

$100^{-1/2} = 1/10$



Some more functions... $2^{\bullet}$ expr. of x: 2^x

Change to: $e^{\bullet}$

$c \doteq 3.321928095$

$2^c = 10$

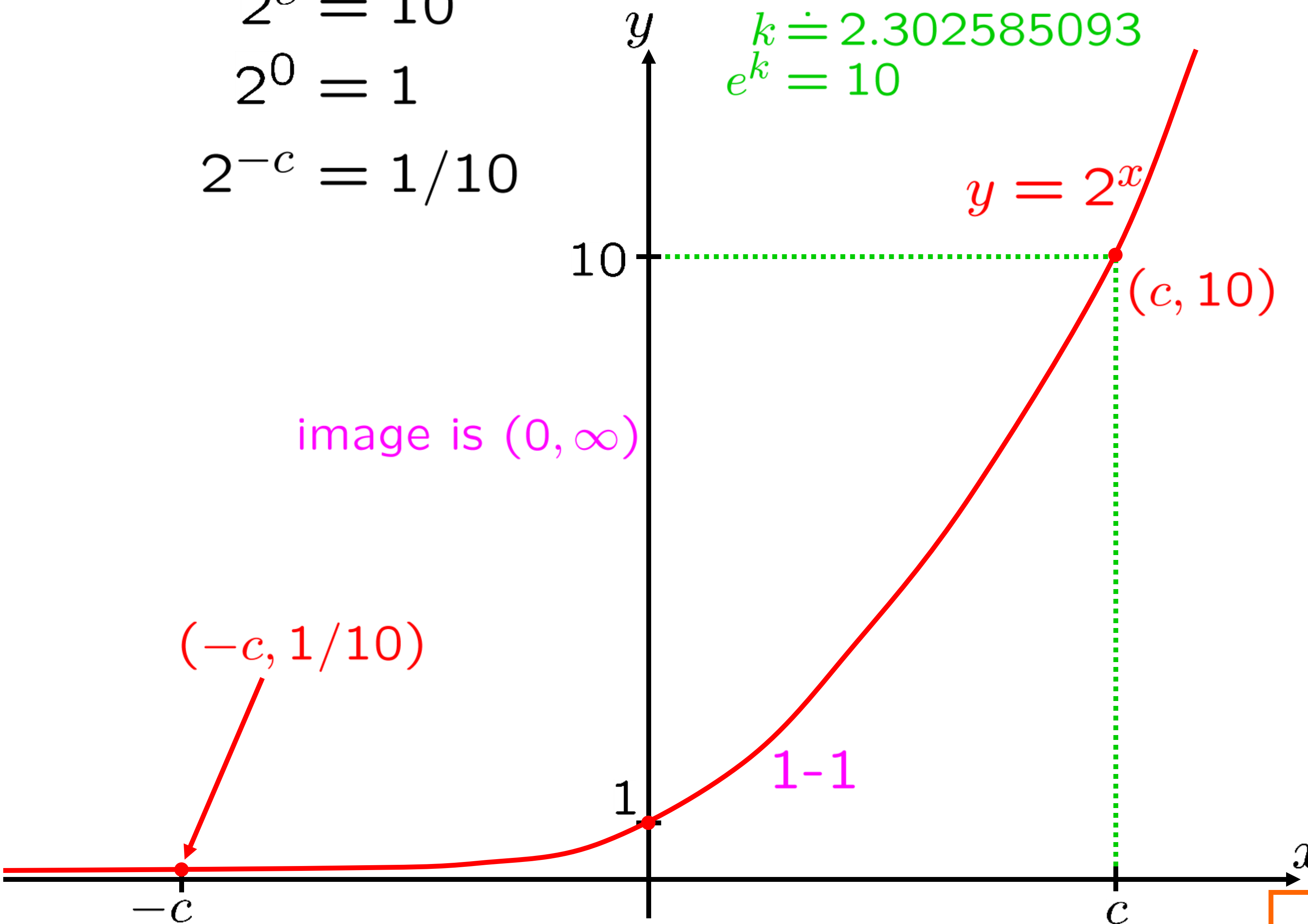
$k \doteq 2.302585093$

$e^k = 10$

$2^c = 10$

$2^0 = 1$

$2^{-c} = 1/10$



domain is $\mathbb{R} = (-\infty, \infty)$

Some more functions... $e^{\bullet}$ expr. of x: e^x

Change to: $(1/10)^{\bullet}$

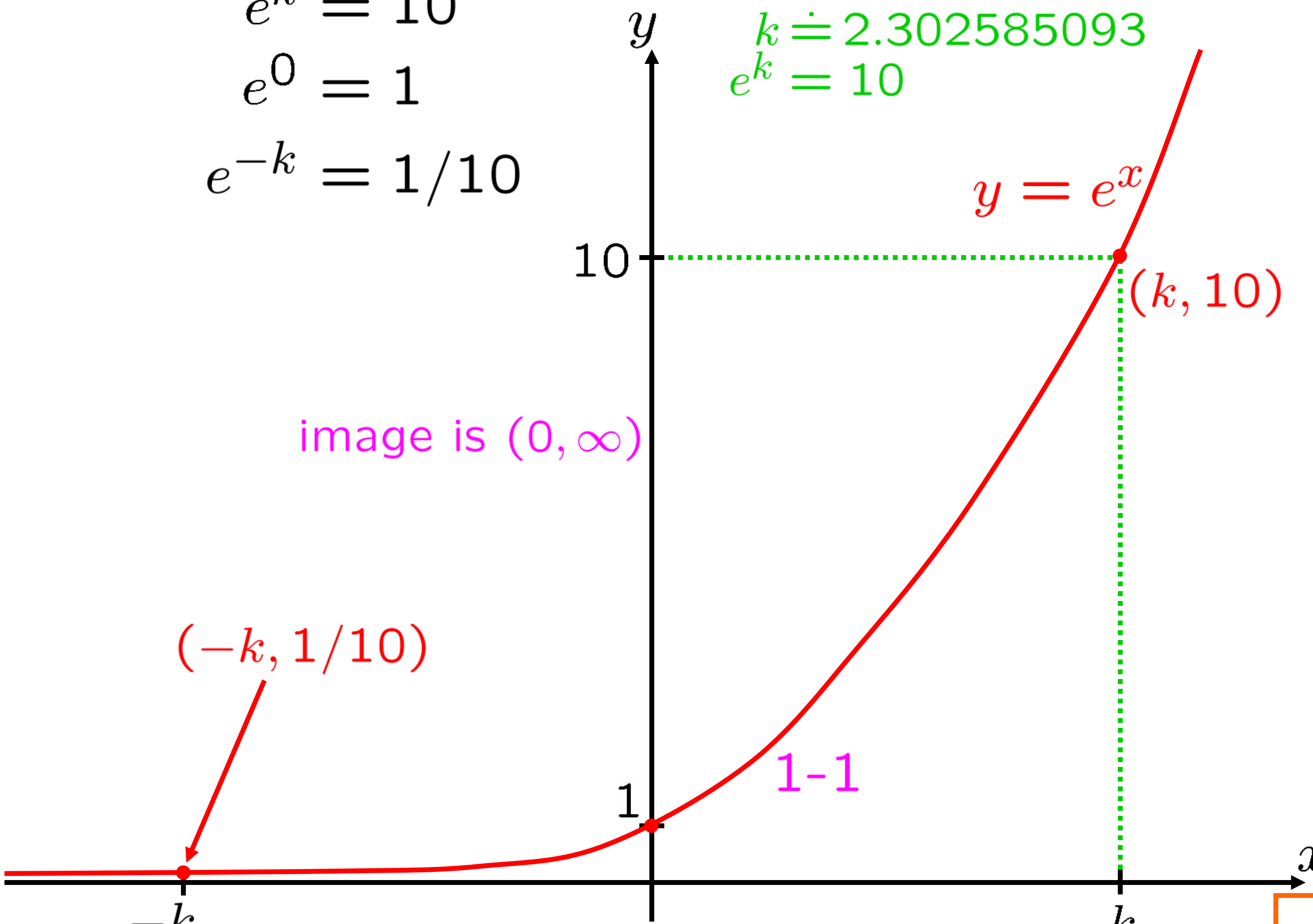
$u = -1$

$(1/10)^u = 10$

$$e^k = 10$$

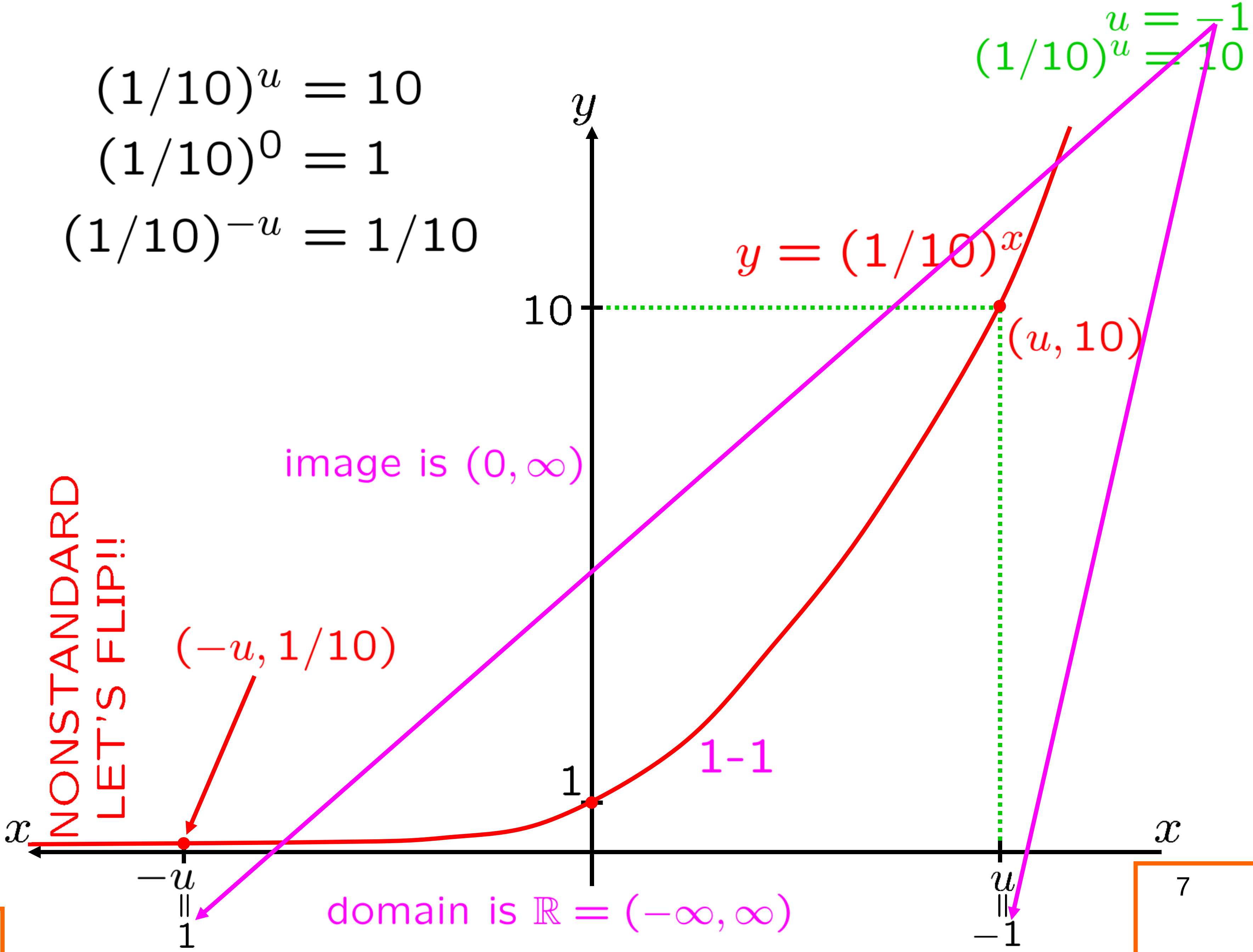
$$e^0 = 1$$

$$e^{-k} = 1/10$$



domain is $\mathbb{R} = (-\infty, \infty)$

$(1/10)^u = 10$
$(1/10)^0 = 1$
$(1/10)^{-u} = 1/10$



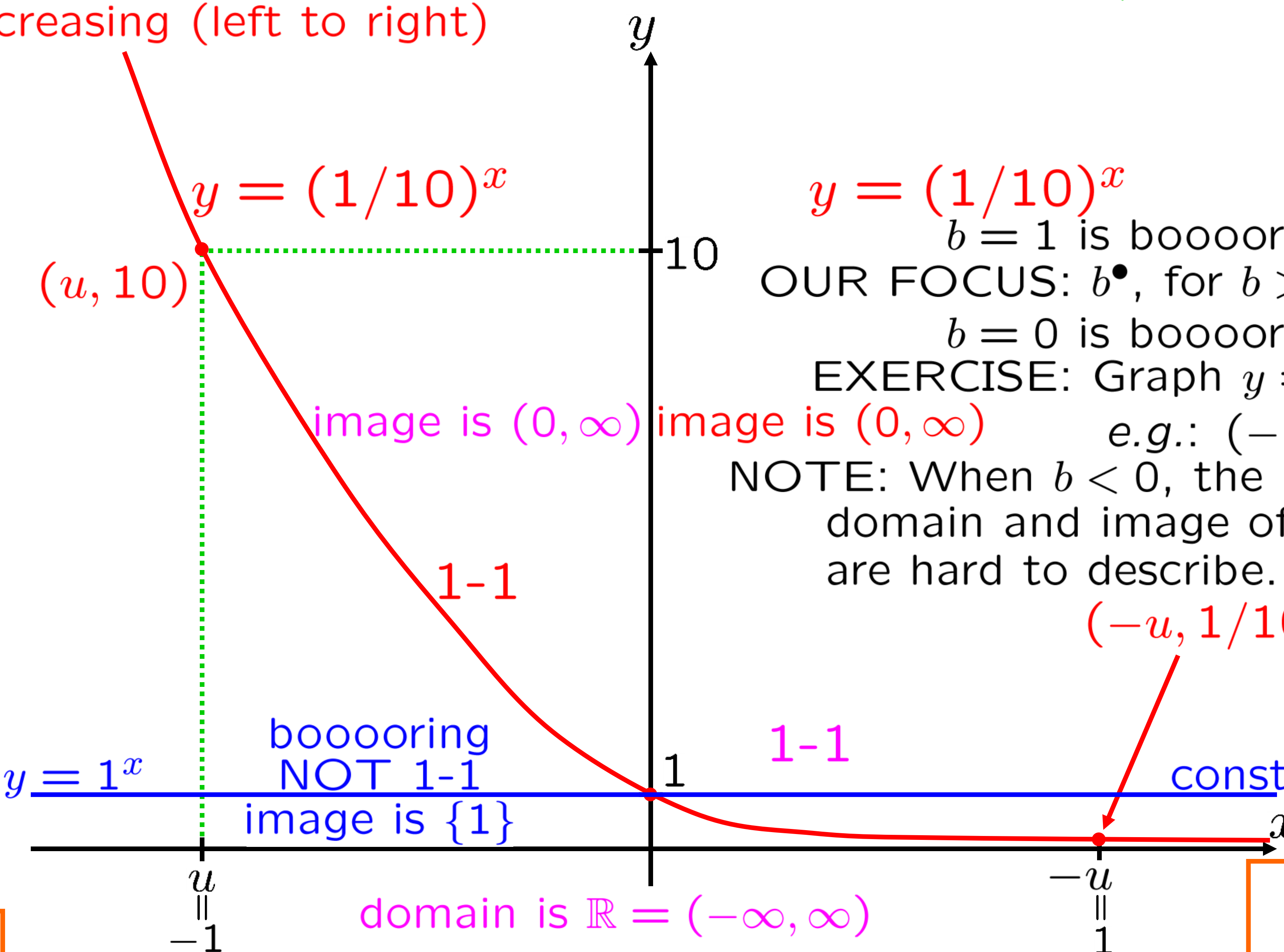
Some more functions. . . $(1/10)^\bullet$ expr. of x: $(1/10)^x$

$1^\bullet = 1$

$u = -1$

$(1/10)^u = 10$

Whenever $0 <$ base < 1:
decreasing (left to right)



$y = (1/10)^x$

$b = 1$ is booooring!

OUR FOCUS: $b^\bullet$, for $b > 0$

$b = 0$ is booooring!

EXERCISE: Graph $y = 0^x$.

e.g.: $(-1)^x$

NOTE: When $b < 0$, the domain and image of b^x are hard to describe.

§4.6

Some more functions. . .

10^{100} has 100 zeroes

$\forall b > 1$: FAST
increasing exponential fn
"exponential increase"

$\forall b \in (0,1)$: FAST
decreasing exponential fn
"exponential decay"

$(1/10)^\bullet$ expr. of x: $(1/10)^x$

$e^\bullet$ expr. of x: e^x

$2^\bullet$ expr. of x: 2^x

$100^\bullet$ expr. of x: 100^x

$10^\bullet$ expr. of x: 10^x

$\forall b \in (0,\infty)$: $b^\bullet$ expr. of x: b^x

Some more *INVERSE* functions. . .

$\forall b \in (0,\infty)\backslash\{1\}$: $\log_b := [\text{inverse of } b^\bullet] = (b^\bullet)^{-1}$

expr. of x: $\log_b x$

$\forall b > 1$: SLOW
increasing logarithmic fn
"logarithmic increase"

$\log_{10}(10^{100}) = 100$

$\forall b \in (0,1)$: SLOW
decreasing logarithmic fn
"logarithmic decay"

$\ln := \log_e$

$\ln x = \log_e x$

Some more functions. . .

$\forall b > 1$:
increasing exponential fn
"exponential increase"

$\forall b \in (0,1)$:
decreasing exponential fn
"exponential decay"

$\forall b > 0$: $b^\bullet$
expr. of x: b^x

image is $(0, \infty)$

$b^\bullet : \mathbb{R} \to \mathbb{R}$

1-1, if $b > 0$ and $b \neq 1$

Some more *INVERSE* functions. . .

$\forall b > 1$:
increasing logarithmic fn
"logarithmic increase"

$\forall b \in (0,1)$:
decreasing logarithmic fn
"logarithmic decay"

$\forall b > 0$: $\log_b := [\text{inverse of } b^\bullet]$

expr. of x: $\log_b x$

$\ln := \log_e$

$\ln x = \log_e x$

$\forall b > 0$: $b^\bullet$ expr. of x: b^x
$\log_b := [\text{inverse of } b^\bullet]$
expr. of x: $\log_b x$
$\forall b > 1$: increasing logarithmic fn "logarithmic increase"
$\ln := \log_e$
$\ln x = \log_e x$

Some more functions. . .

$\forall b > 1$:
increasing exponential fn
"exponential increase"

$\forall b \in (0,1)$:
decreasing exponential fn
"exponential decay"

$\forall b > 0$: $b^\bullet$
expr. of x: b^x

$b^\bullet : \mathbb{R} \to (0,\infty)$
1-1, if $b > 0$ and $b \neq 1$
and onto $(0,\infty)$

Some more *INVERSE* functions. . .

$\forall b > 1$:
increasing logarithmic fn
"logarithmic increase"

$\forall b \in (0,1)$:
decreasing logarithmic fn
"logarithmic decay"

$\forall b > 0$:
$\log_b :=$ [inverse of $b^\bullet$]
expr. of x: $\log_b x$
$\log_b : (0,\infty) \to \mathbb{R}$

$\ln := \log_e : (0,\infty) \to \mathbb{R}$
$\ln x = \log_e x$

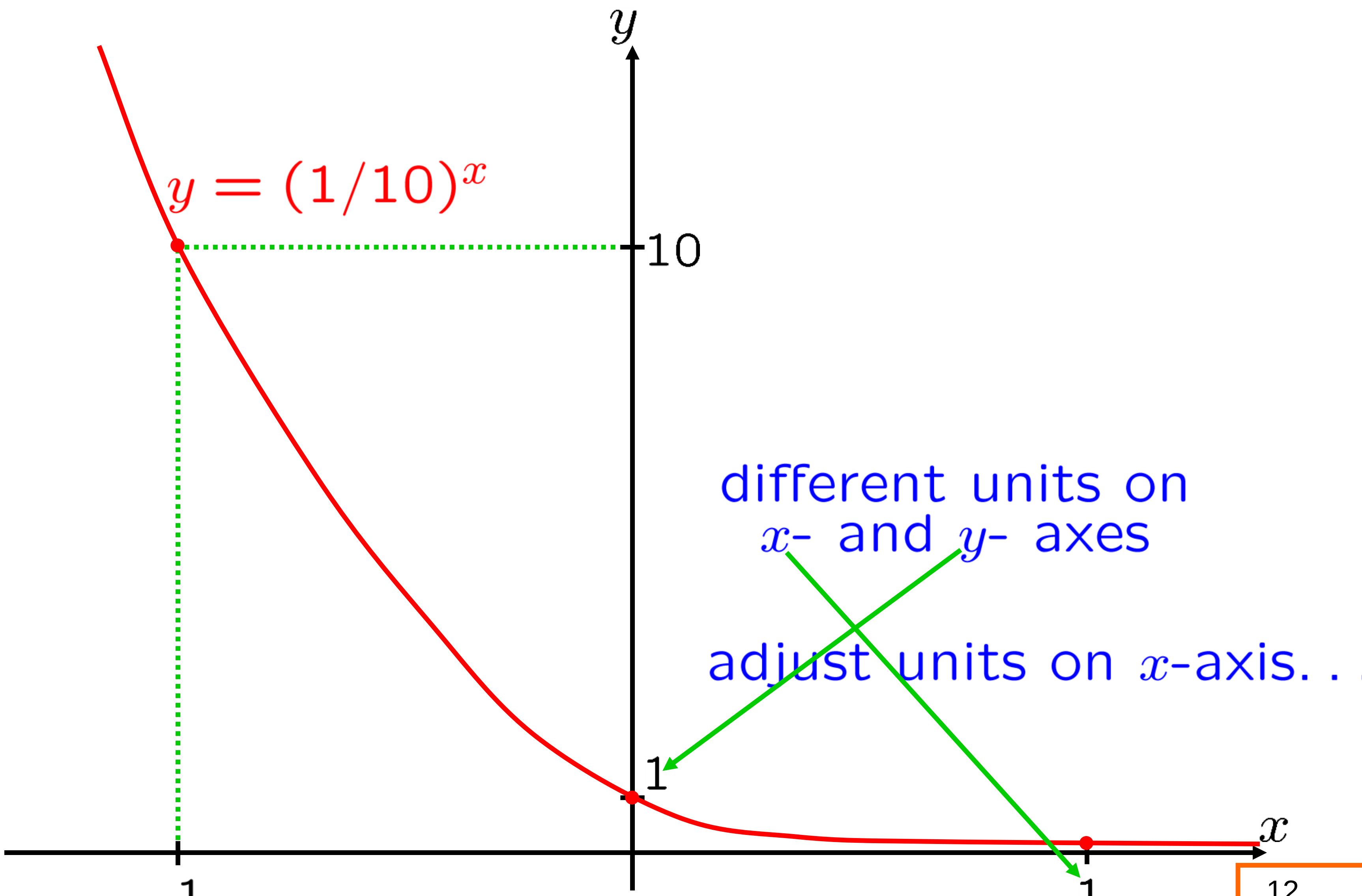
SKILL
dom, im of $b^\bullet$, $\log_b$

$\log_b(b^x) = x$, $\forall x \in \mathbb{R}$, $\forall b \in (0,\infty)\backslash\{1\}$
$b^{\log_b x} = x$, $\forall x > 0$, $\forall b \in (0,\infty)\backslash\{1\}$

$b :\to e$

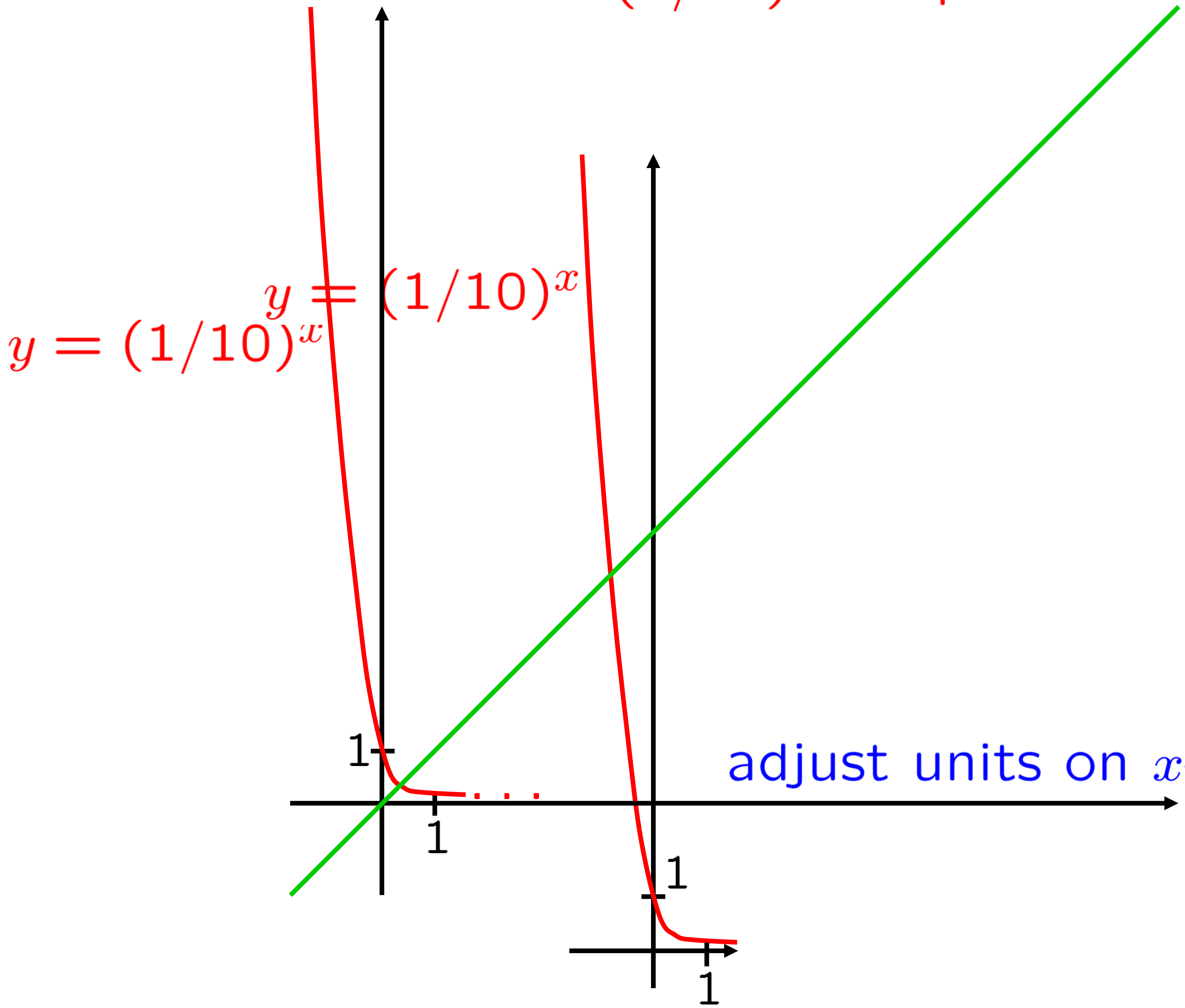
$\ln(e^x) = x$, $\forall x \in \mathbb{R}$
$e^{\ln x} = x$, $\forall x > 0$

Next subtopic:
Gphs of log fns



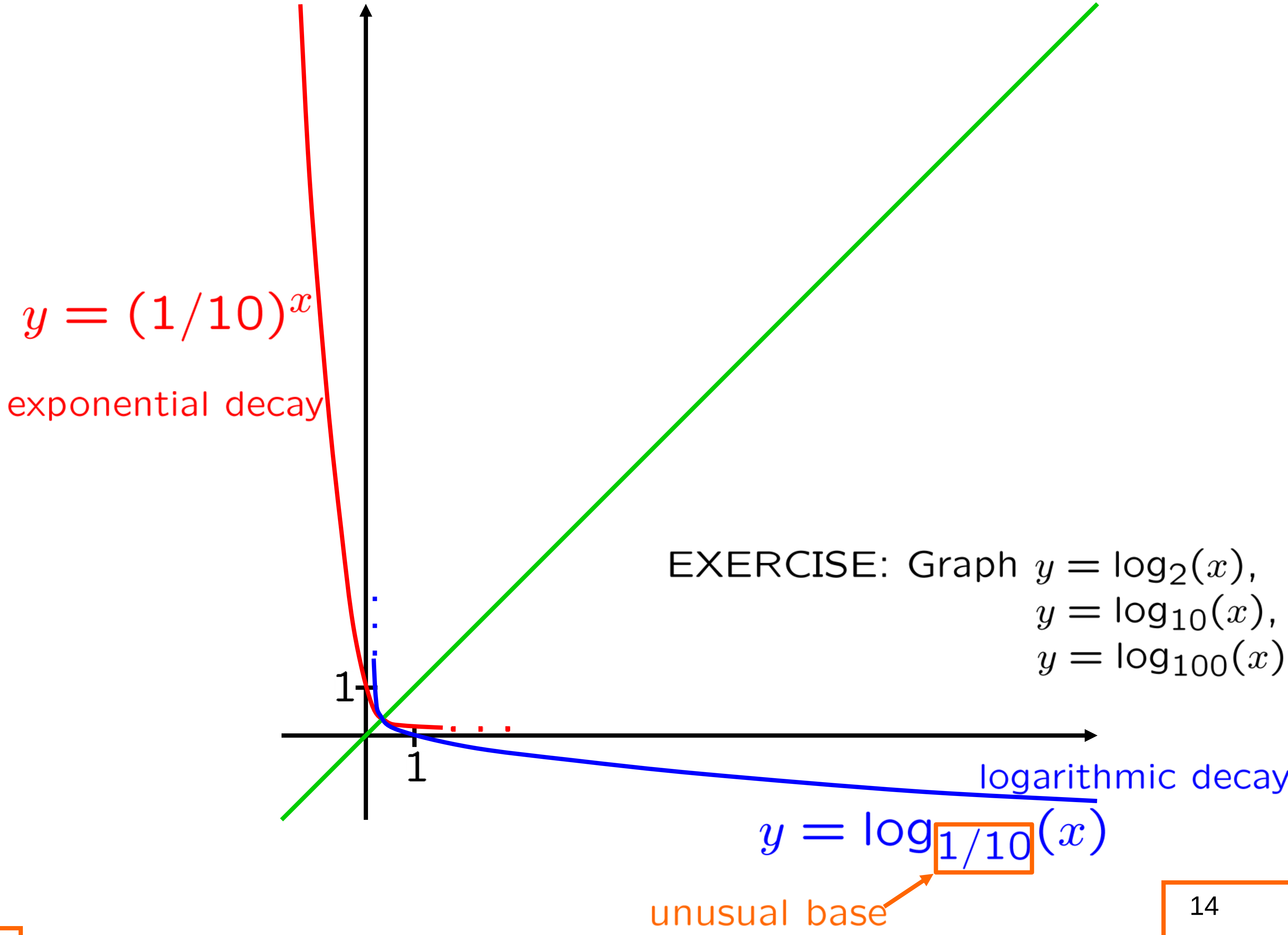
y
$y = (1/10)^x$
10
1
x
-1
1
different units on
x- and y- axes
adjust units on x-axis. . .

$(1/10)^{\bullet}$ expr. of x: $(1/10)^x$



adjust units on x-axis. . .

Next, let's graph $y = \ln(x)$.



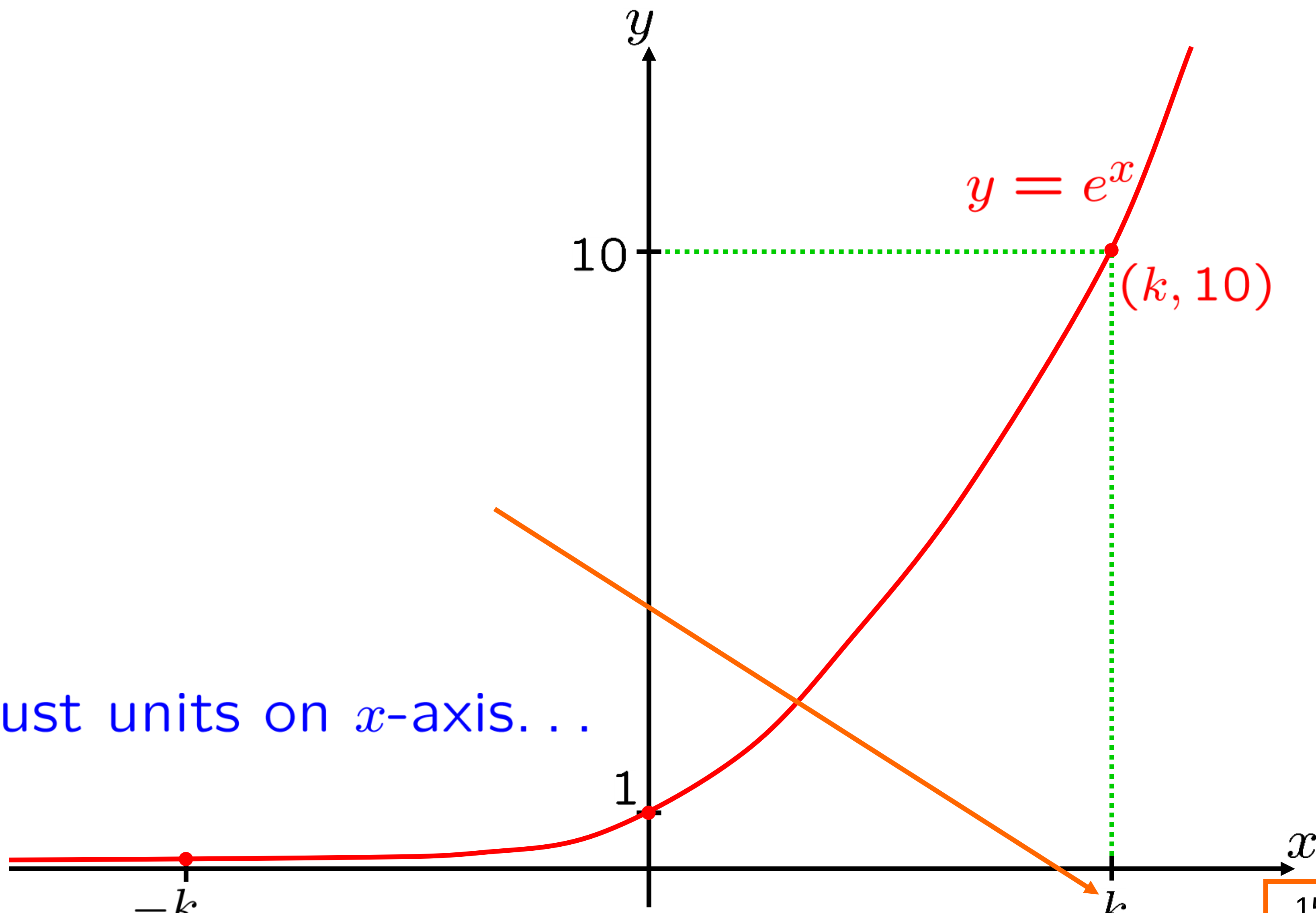
EXERCISE: Graph $y = \log_2(x)$,
$y = \log_{10}(x)$,
$y = \log_{100}(x)$.

Next, let's graph $y = \ln(x)$.

$e^{\bullet}$ expr. of x: e^x

$k \doteq 2.302585093$

$e^k = 10$



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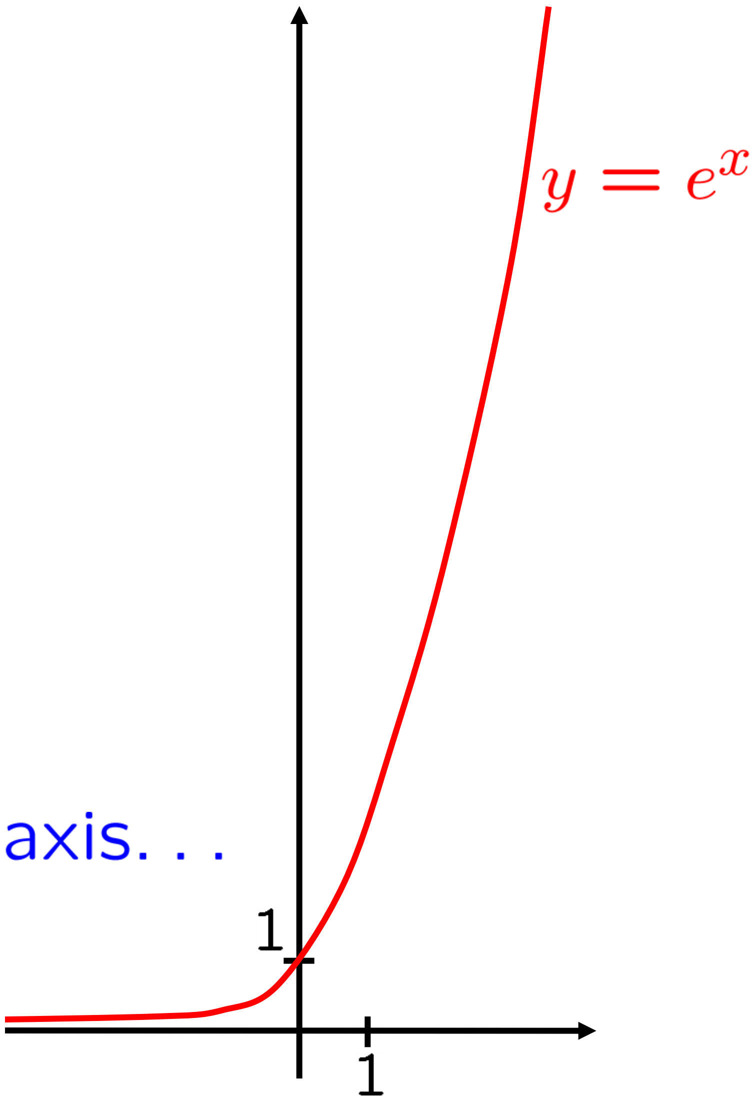
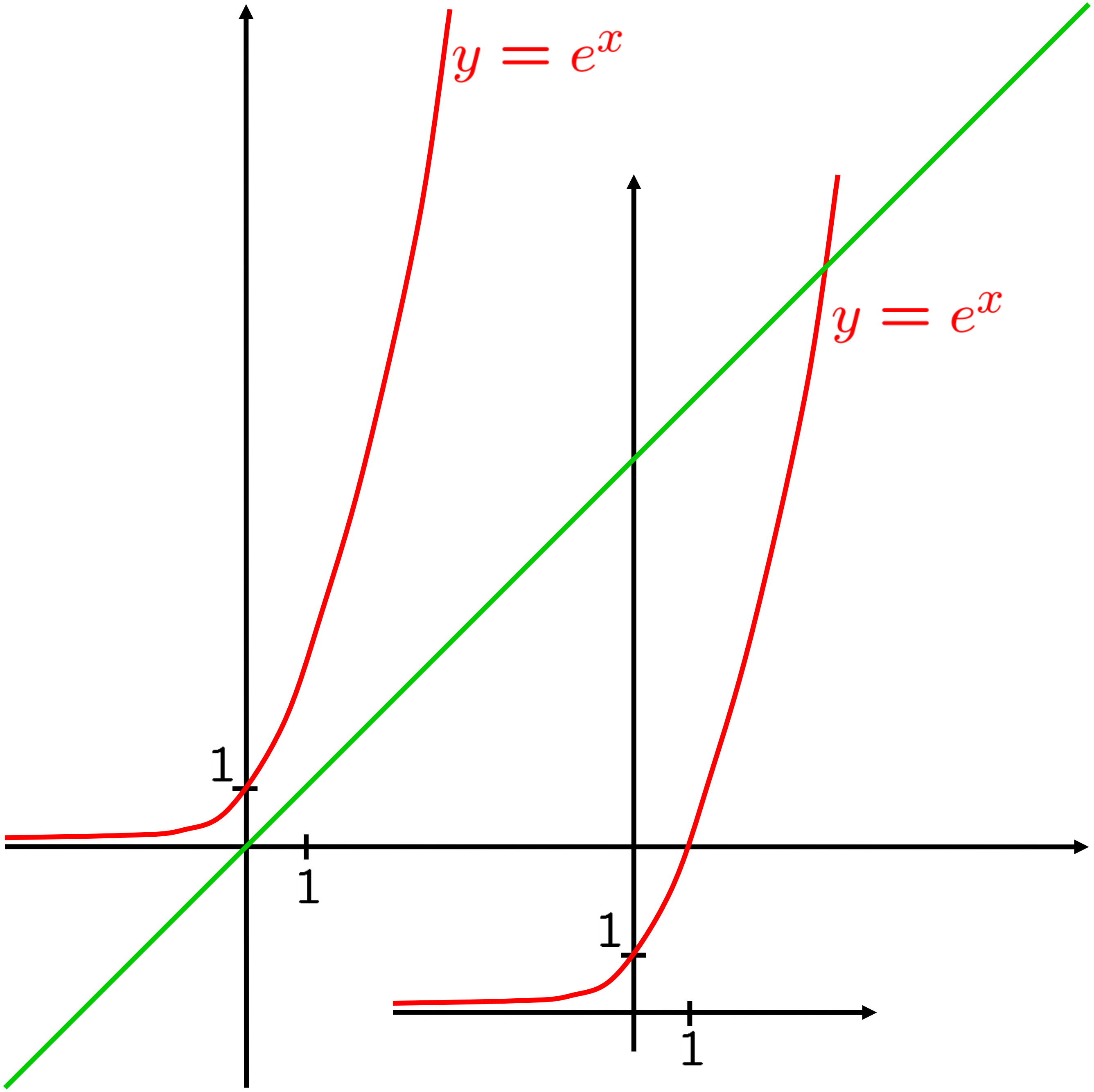
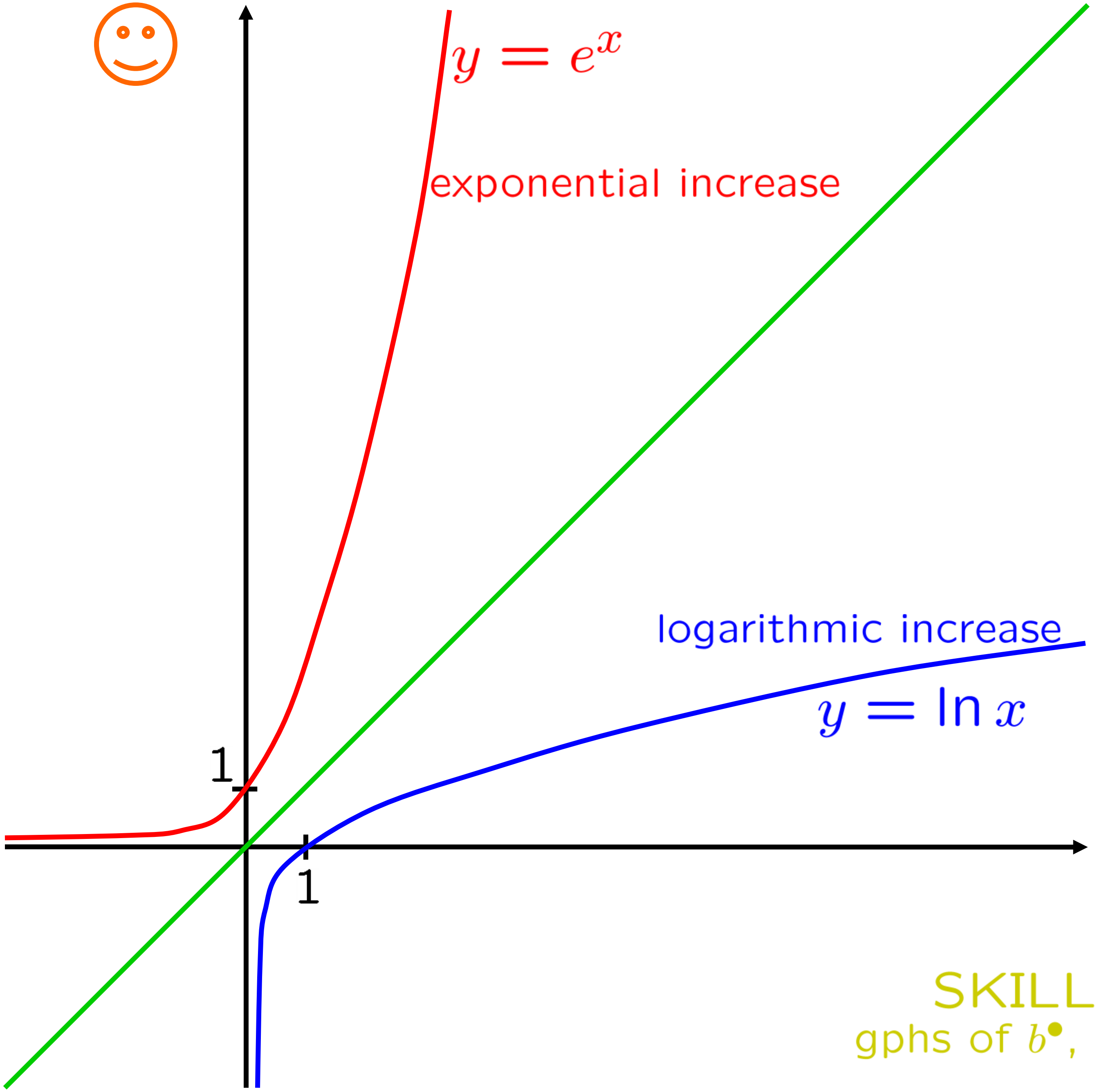


adjust units on x-axis. . .

Next, let's graph $y = \ln(x)$.



Next, let's graph $y = \ln(x)$.



SKILL
gphs of $b^\bullet$, $\log_b$

Exercise: Use the Intermediate Value Theorem to show that there is a root of the following equation in the interval $(1,2)$.

$$\ln x = e^{-3x}$$

STOP

SKILL
root via IVT

Pf: $f(x) := (\ln x) - e^{-3x}$

f is continuous on $[1,2]$

$$f(1) = (\ln 1) - e^{-3} \doteq 0 - 0.04979 < 0$$

$$f(2) = (\ln 2) - e^{-6} \doteq 0.6931 - 0.04979 > 0$$

0 is between $f(1)$ and $f(2)$.

IVT: $\exists c \in (1,2)$ s.t. $f(c) = 0$

$$\| $$

$$(\ln c) - e^{-3c}$$

$$\ln c = e^{-3c}$$ QED