CALCULUS
Antidifferentiation

Key point: Differentiation is about splitting things apart into small pieces.
"Integration" is about putting the pieces back together to form the whole.

$$(f(1)) - (f(0)) = \left(\begin{array}{l} \quad [(f(0.01)) - (f(0))] \\ +[(f(0.02)) - (f(0.01))] \\ +\cdots \\ +[(f(0.99)) - (f(0.98))] \\ +[(f(1.00)) - (f(0.99))] \end{array} \right)$$

Terms all small.
Renormalize,
by dividing by 0.01.

We can reassemble $(f(1)) - (f(0))$
from the pieces,
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We can reassemble $(f(1)) - (f(0))$ from the "renormalized" pieces, via "renormalized" addition.

$$\frac{(f(0.76)) - (f(0.75))}{0.01} = \left[\frac{(f(0.75+h)) - (f(0.75))}{h}\right]_{h:\to 0.01}$$

$$\approx \lim_{h\to 0} \frac{(f(0.75+h)) - (f(0.75))}{h}$$

$$= f'(0.75)$$

§7.1

Key point: Differentiation is about splitting things apart into small pieces.
"Integration" is about putting the pieces back together to form the whole.

Going from f' to f is called "antidifferentiation" . . .

Integration tells us how to
reassemble $(f(1)) - (f(0))$
from all the $f'(x)$, with $x \in [0,1]$.

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§7.1

Let $D \subseteq \mathbb{R}$. Let $f : D \to \mathbb{R}$ be a function.

cf. §7.2, p. 145 DEFINITION

A function $F : D \to \mathbb{R}$ is called an **antiderivative of** f if, $\forall x \in D$, we have: $F'(x) = f(x)$.

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e.g.: $f(x) = x^2$

Guess: $F(x) = \frac{1}{3}x^3$

$F'(x) = x^2 = f(x)$

~~Guess: $F(x) = x^3$~~

~~$F'(x) = 3x^2 \neq f(x)$~~

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Guess: $F(x) = \frac{1}{3}x^3$ ☺

$F'(x) = x^2 = f(x)$

Guess: $F(x) = \frac{1}{3}x^3 + 6$ ☺

$F'(x) = x^2 = f(x)$

Other antiderivatives: (of x^2 w.r.t. x)

$\frac{1}{3}x^3 + 8$

$\frac{1}{3}x^3 + 3$

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d/dx is not "1-1"
and so is not invertible.

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cf. §6.5, p. 136 (TH'M 6.28):

If $g'(x) = h'(x)$, for all x in an interval I, then $g - h$ is constant on I; that is, $\exists c \in \mathbb{R}$ s.t. $\forall x \in I$,

$$g(x) = (h(x)) + c.$$

works for any kind of interval (open, closed, half-open) (bdd, unbdd)

---

e.g.: $f(x) = x^2$ $\mathrm{dom}[f] = \mathbb{R}$ is an interval.

Guess: $F(x) = \frac{1}{3}x^3$ ☺

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$\{\frac{1}{3}x^3 + C \,|\, C \in \mathbb{R}\}$ is the *set* of *all* antiderivatives of x^2 w.r.t. x.

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Traditional to drop the set braces and everything after the vertical line (|)

More on this later . . .

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$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1 - x^2}}$$

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domain: $-1 < x < 1$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1 - x^2}}$$

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domain: $-1 \le x \le 1$

An antiderivative of $\dfrac{1}{\sqrt{1-x^2}}$ w.r.t. x is

$\arcsin x$, $\quad -1 < x < 1$. sloppy...

RESTRICTION OF FUNCTIONS

arcsin

$\arcsin | (-1, 1)$

RESTRICTIONS OF EXPRESSIONS

$\arcsin x$

$\arcsin x$, $\quad -1 < x < 1$

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An antiderivative of $\dfrac{1}{\sqrt{1-x^2}}$ w.r.t. x is

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cf. §7.2, p. 145 DEFINITION

Let $a, b \in \mathbb{R}$ satisfy $a < b$. Assume that $(a, b) \subseteq D$.

A function F is called an **antiderivative of** f **on** (a, b) if, $\forall x \in (a, b)$, we have: $F'(x) = f(x)$.

$\arcsin x$ is an antiderivative w.r.t. x

of $\dfrac{1}{\sqrt{1-x^2}}$ on $-1 < x < 1$.

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Let $a, b \in \mathbb{R}$ satisfy $a < b$. Assume that $[a, b] \subseteq D$.

A function F is called an **antiderivative of** f **on** $[a, b]$

if both F is continuous on $[a, b]$

and F is an antiderivative of f on (a, b).

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contin. on $-1 \leq x \leq 1$

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cf. §7.2, p. 149 The *set* of *all* antiderivatives of $f(x)$ w.r.t. x

e.g.: $\int x^2\,dx = \frac{x^3}{3} + C$

is denoted $\int f(x)\,dx$.

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e.g.: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, if n is positive

$n = -1$??

$n = -1/2$??

e.g.: $\int x^{-1/2}\,dx \underset{\text{sloppy}}{=} \frac{x^{1/2}}{1/2} + C,\ x \neq 0$

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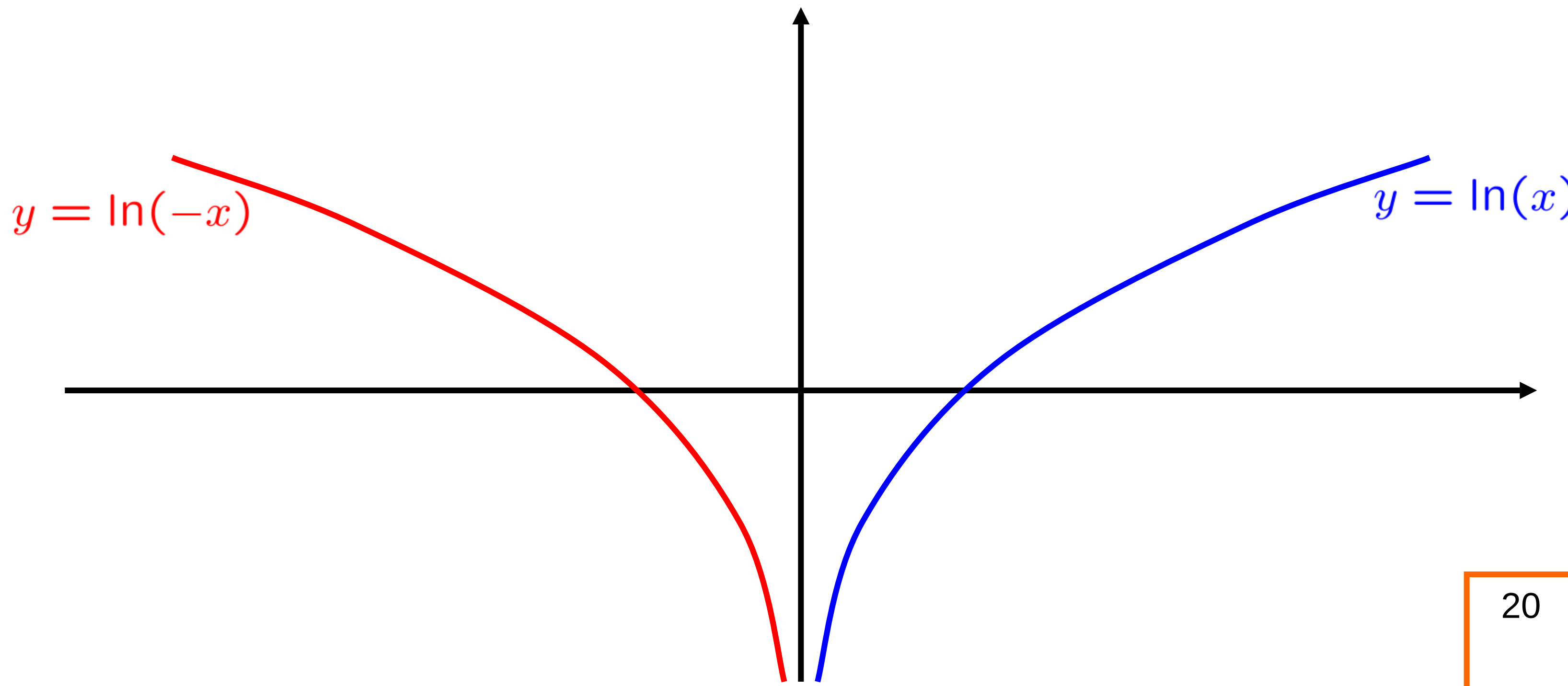

Spp

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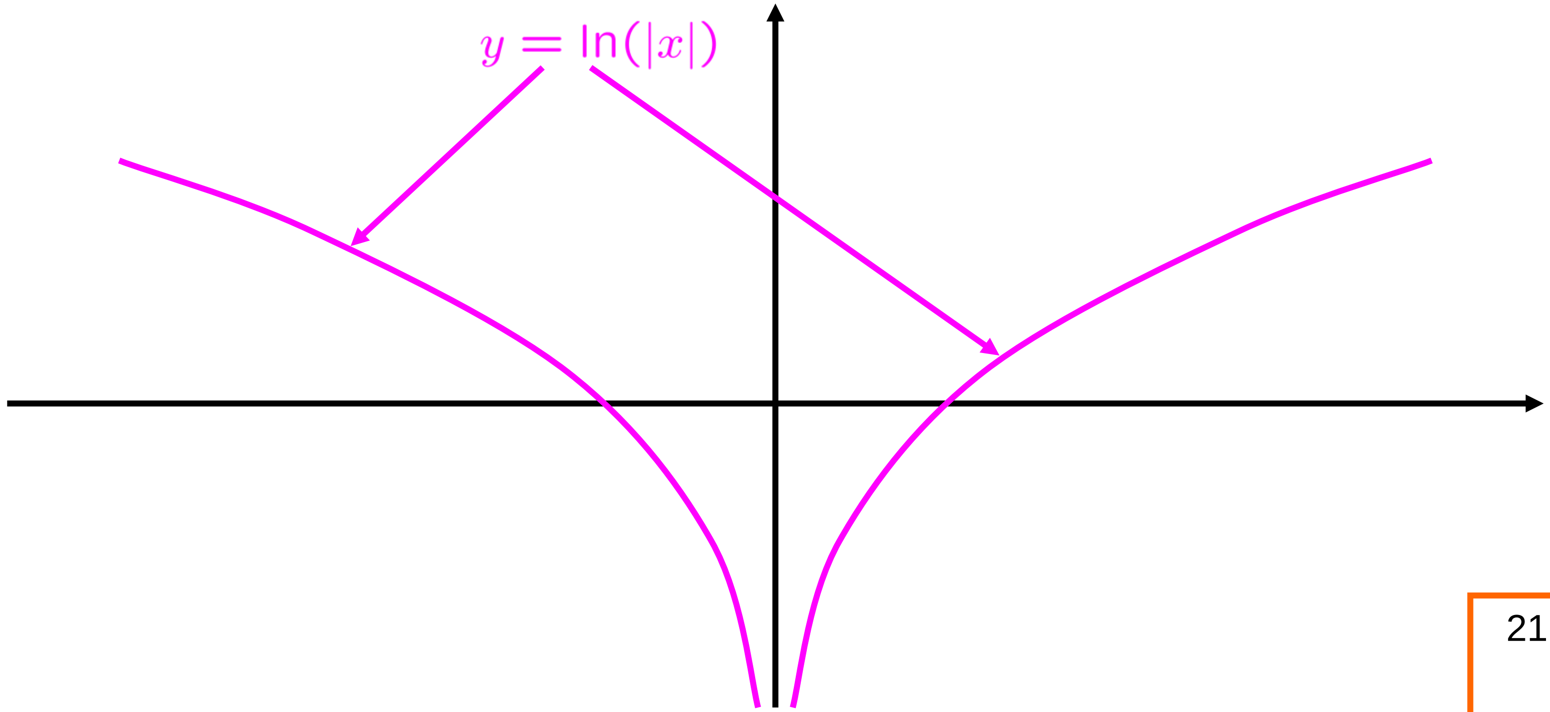
$[\ln(|x|)] + C$??

$\ln(|x|)$ is an antiderivative of $\frac{1}{x}$ w.r.t. x.



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cf. §6.5, p. 136 (TH'M 6.28):

If $g'(x) = h'(x)$, for all x in an interval I,

then $g - h$ is constant on I;

that is, $\exists c \in \mathbb{R}$ s.t. $\forall x \in I$,

$$g(x) = (h(x)) + c.$$

works for any kind of interval (open, closed, half-open) (bdd, unbdd)

domain: $x \neq 0$ NOT an interval

e.g.: $\int \frac{1}{x} dx = $??

~~$[\ln(|x|)] + C$~~ ??

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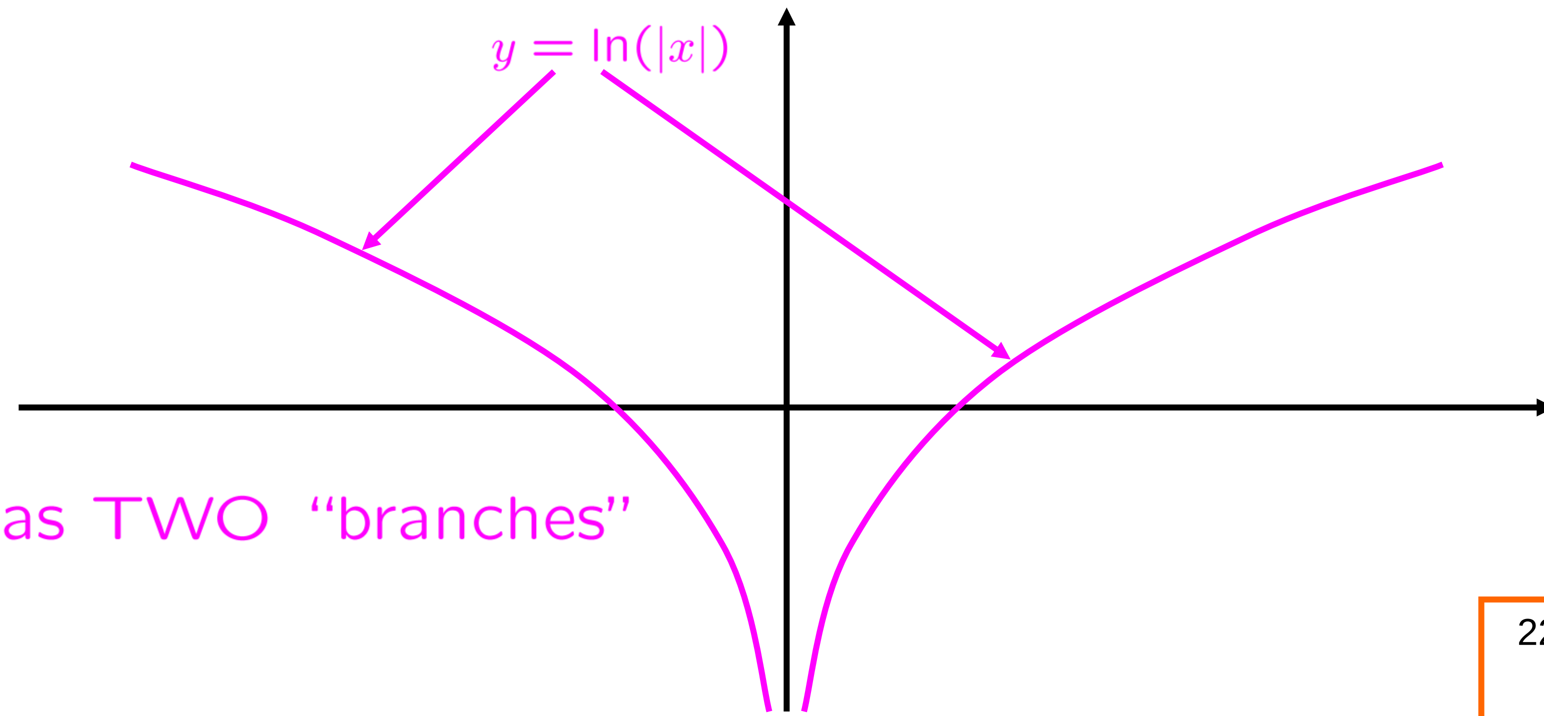

graph has TWO "branches"

Spp

cf. §6.5, p. 136 (TH'M 6.28):

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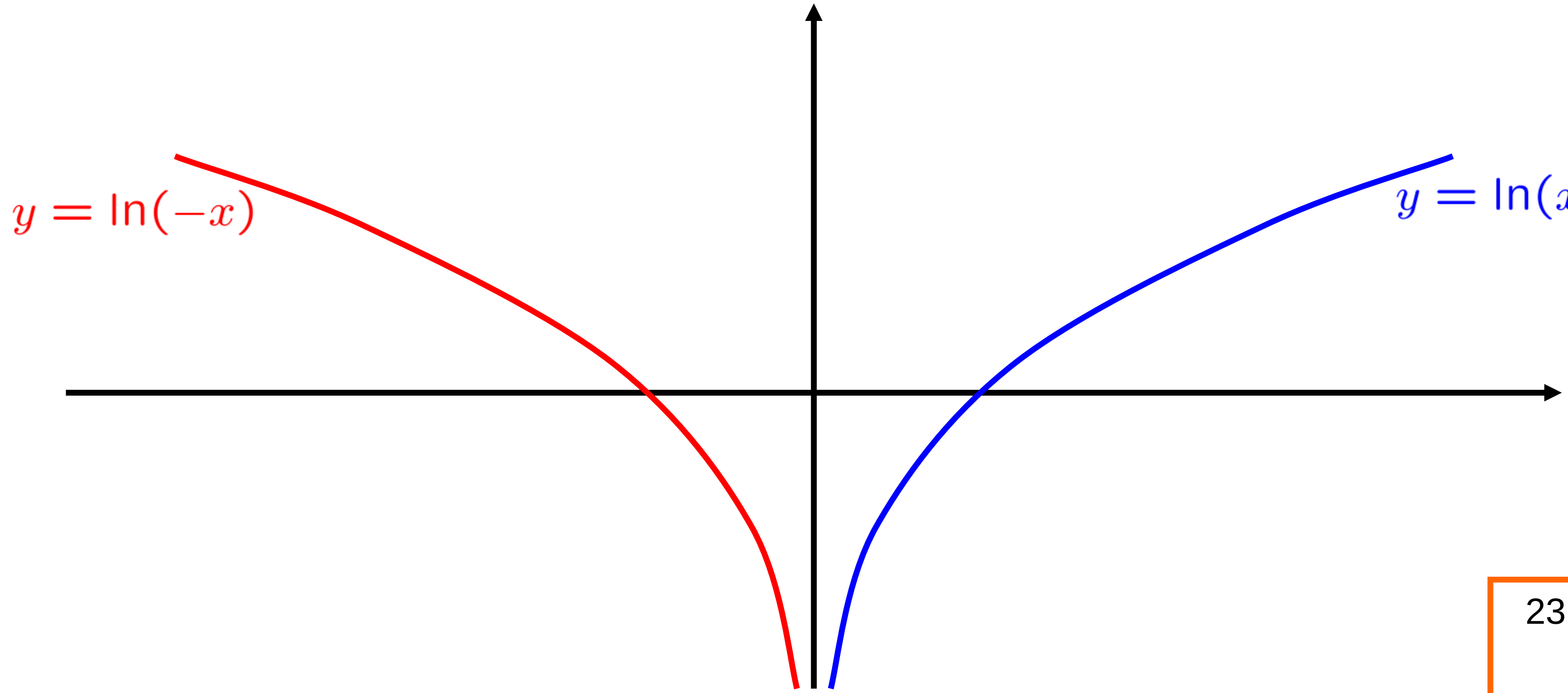

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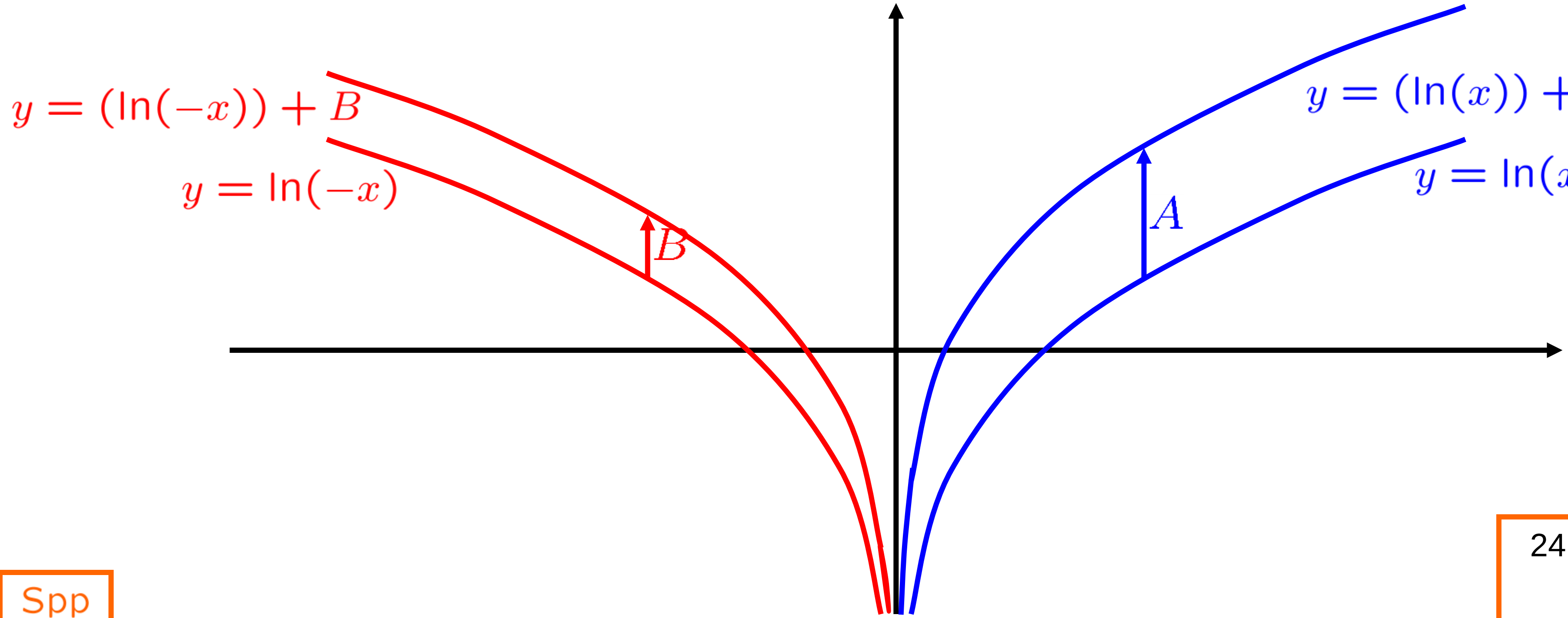
e.g.: $\int \frac{1}{x} dx = ??$

$$\ln_{AB}(x) := \begin{cases} (\ln(x)) + A, & \text{if } x > 0 \\ (\ln(-x)) + B, & \text{if } x < 0 \end{cases}$$



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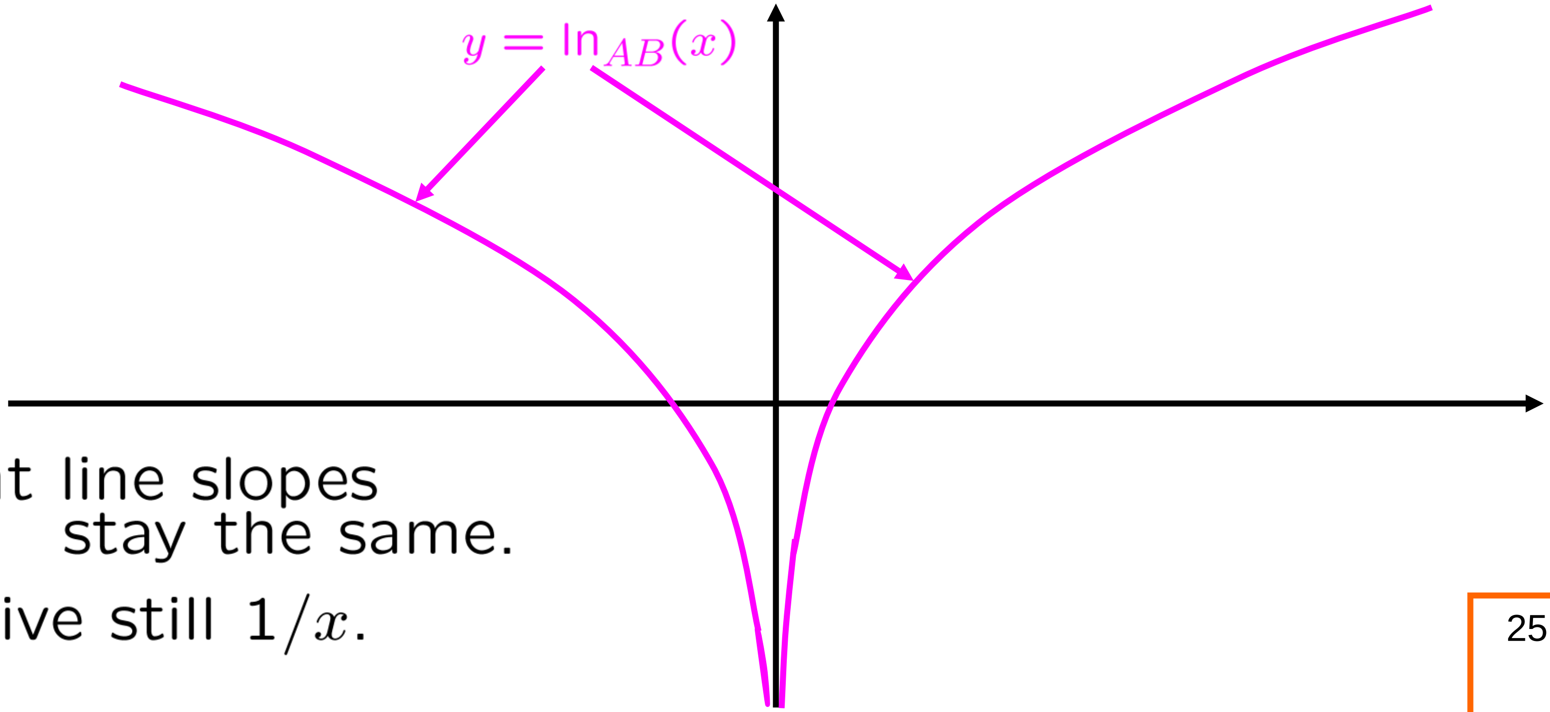

Tangent line slopes
stay the same.

Derivative still $1/x$.

$\forall A, B \in \mathbb{R},$

$\ln_{AB}(x)$ is an antiderivative of $\frac{1}{x}$ w.r.t. x.

$$\int \frac{1}{x}\,dx = \{\ln_{AB}(x) \mid A, B \in \mathbb{R}\}$$ too complicated...

e.g.: $\int \frac{1}{x}\,dx = ??$

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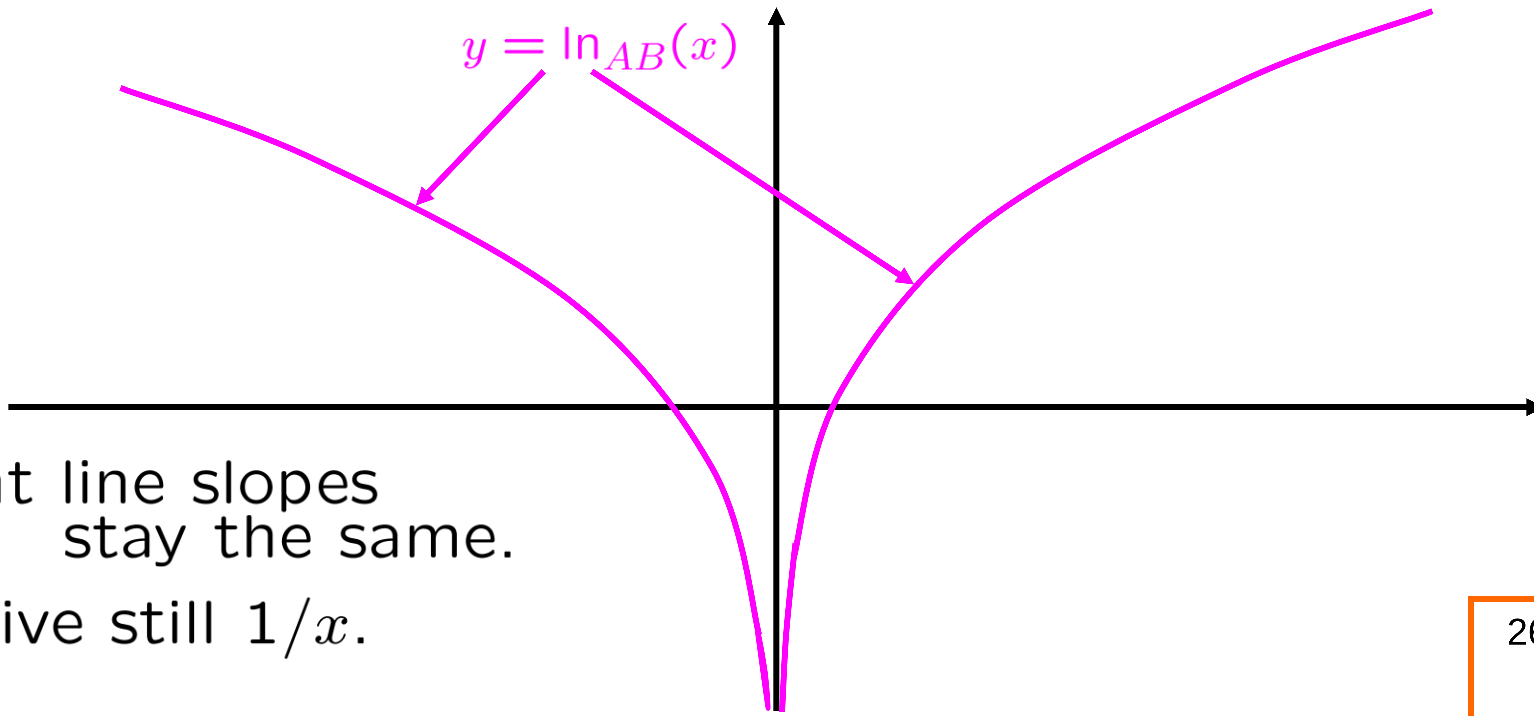

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$$\int \frac{1}{x}\,dx \overset{\text{sloppy}}{=} [\text{ln}(|x|)] + C$$

It's quite common to list one antiderivative "plus C", even in cases where it's technically wrong!

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad \text{if } n \neq -1$$

sometimes sloppy

$$\int x^{-1/3}\,dx \underset{\substack{\text{sloppy}\\ \times 2}}{=} \frac{x^{2/3}}{2/3} + C, \quad x \neq 0$$

differing domains and ...
domain not an interval

$$\int x^{-2}\,dx \underset{\text{sloppy}}{=} \frac{x^{-1}}{-1} + C$$

domain not an interval

$$\int x^{-1/2}\,dx \underset{\text{sloppy}}{=} \frac{x^{1/2}}{1/2} + C$$

differing domains

$$\int \frac{1}{x}\,dx \overset{\text{sloppy}}{=} [\ln(|x|)] + C$$

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differing domains and ...
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$$\int x^{-2}\,dx \underset{\text{sloppy}}{=} \frac{x^{-1}}{-1} + C$$

domain not an interval

Don't worry about
all this sloppiness, ...

$$\int x^{-1/2}\,dx \underset{\text{sloppy}}{=} \frac{x^{1/2}}{1/2} + C$$

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$$\int \frac{1}{x}\,dx = [\ln(|x|)] + C$$

$$\int x^{n}\,dx = \frac{x^{n+1}}{n+1} + C, \quad \text{if } n \neq -1$$

Don't worry about all this sloppiness, . . .
but the following considered important. . .

Next: table of antiderivatives

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative
$c(f(x))$	$c(F(x))$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$
$x^n, \quad n \neq -1$	$\frac{x^{n+1}}{n+1}$ sometimes sloppy
$1/x$	$\ln(\lvert x \rvert)$
e^x	e^x

Function	Particular antiderivative
$\cos x$	$\sin x$
$\sec^2 x$	$\tan x$
$(\sec x)(\tan x)$	$\sec x$
COMPLEMENTARY FORMULAS	
$\sin x$	$-\cos x$
$\csc^2 x$	$-\cot x$
$(\csc x)(\cot x)$	$-\csc x$

Assume $F' = f$ $G' = g$.

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative	Function	Particular antiderivative
$c(f(x))$	$c(F(x))$	$\cos x$	$\sin x$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
$x^n, \quad n \neq -1$	$\frac{x^{n+1}}{n+1}$ sometimes sloppy	$(\sec x)(\tan x)$	$\sec x$
$1/x$	$\ln(\lvert x \rvert)$	$\frac{1}{\sqrt{1-x^2}}$	$-\cos x$
e^x	e^x		
$\sin x$	$-\cos x$		

Assume
$F' = f$
$G' = g$.

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative	Function	Particular antiderivative
$c(f(x))$	$c(F(x))$	$\cos x$	$\sin x$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
		$(\sec x)(\tan x)$	$\sec x$
$x^n, \quad n \neq -1$	$\frac{x^{n+1}}{n+1}$ sometimes sloppy	$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$, $-1 < x < 1$
$1/x$	$\ln(\lvert x \rvert)$	$\frac{1}{1+x^2}$	$\arctan x$
e^x	e^x		
$\sin x$	$-\cos x$		

Assume $F' = f$ $G' = g$.

EXAMPLE: Find all functions g such that

$$g'(x) = 3\cos x + \frac{2x^4 - \sqrt[3]{x}}{x^7}.$$

$$g'(x) = 3\cos x + 2x^{-3} - x^{-20/3}$$

ANTIDIFF

$$g(x) = 3\sin x + 2\left[\frac{x^{-2}}{-2}\right] - \frac{x^{-17/3}}{-17/3} + C$$

~~$+3K \quad +2L \quad -(+M)$~~

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative	Function	Particular antiderivative
$c(f(x))$	$c(F(x))$	$\cos x$	$\sin x$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
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SKILL
all antiderivs

$$g'(x) = 3\cos x + 2x^{-3} - x^{-20/3}$$

$$g(x) = 3\sin x + 2\left[\frac{x^{-2}}{-2}\right] + \frac{x^{-17/3}}{+17/3} + C$$

sloppy

$$= 3\sin x - x^{-2} + 3x^{-17/3}/17 + C$$ ■

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative	Function	Particular antiderivative
$c(f(x))$	$c(F(x))$	$\cos x$	$\sin x$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
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e^x	e^x	$\frac{1}{1+x^2}$	$\arctan x$
$\sin x$	$-\cos x$		

Assume $F' = f$, $G' = g$.

EXAMPLE:

Find f if both $f'(x) = \boxed{e^{2x}} + \dfrac{15}{\sqrt{1-x^2}}$ and $f(0) = -2$.

ANTIDIFF

$$\frac{e^{2x}}{2}$$

$$\frac{d}{dx}\left[e^{2x}\right] = \left[e^{2x}\right][2]$$

TABLE OF ANTIDIFFERENTIATION FORMULAS

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$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
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e^x	e^x		
$\sin x$	$-\cos x$		

Assume $F' = f$, $G' = g$.

EXAMPLE:

Find f if both $f'(x) = e^{2x} + \frac{15}{\sqrt{1-x^2}}$ and $f(0) = -2$.

ANTIDIFF

$$\forall x \in (-1, 1), \quad f(x) = \frac{e^{2x}}{2} + 15 \arcsin x + C$$

Next: antidifferentiate $f(g(x))$

SKILL
one antideriv

$$= \frac{e^{2x}}{2} + 15 \arcsin x - \frac{5}{2} \blacksquare$$

$$-\frac{4}{2} = -2 = f(0) = \frac{1}{2} + (15)(0) + C = \frac{1}{2} + C \Rightarrow -\frac{5}{2} = C$$

TABLE OF ANTIDIFFERENTIATION FORMULAS

Function	Particular antiderivative	Function	Particular antiderivative
$c(f(x))$	$c(F(x))$	$\cos x$	$\sin x$
$(f(x)) + (g(x))$	$(F(x)) + (G(x))$	$\sec^2 x$	$\tan x$
$x^n, \quad n \neq -1$	$\frac{x^{n+1}}{n+1}$ sometimes sloppy	$(\sec x)(\tan x)$	$\sec x$
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e^x	e^x	$\frac{1}{1+x^2}$	$\arctan x$
$\sin x$	$-\cos x$		

Assume $F' = f$, $G' = g$.

EXAMPLE: Find the set of all antiderivatives of $\sin(4x+7)$.

$$\frac{-\cos(4x+7)}{4} + C \quad \blacksquare$$

SKILL
all antiderivs

$$\frac{d}{dx}[-\cos(4x+7)] = (\sin(4x+7))(4) \not\parallel \sin(4x+7)$$

$(-\cos)' \neq \sin$

$$?F' = f \quad \Rightarrow \quad \frac{d}{dx}\left[\frac{F(ax+b)}{a}\right] = f(ax+b)$$

$$(e^\bullet)' = e^\bullet, \text{ so } \frac{d}{dx}\left[\frac{e^{4x+7}}{4}\right] = e^{4x+7}$$

WARNING: No similar formula for $f(ax^2+bx+c)$.

EXAMPLE: Find the set of all antiderivatives of $(4x+7)^{-1}$.

$$(\ln(|\bullet|))' = (\bullet)^{-1}$$

$$\frac{\ln(|4x+7|)}{4} + C \quad \blacksquare$$

sloppy

SKILL
all antiderivs

§8.1

EXAMPLE: Find the set of all antiderivatives of $\sin(4x+7)$.

$$\frac{-\cos(4x+7)}{4} + C \blacksquare$$

SKILL
all antiderivs

Next:
Motion along a line

$$\frac{d}{dx}[-\cos(4x+7)] = (\sin(4x+7))(4) \not\parallel \sin(4x+7)$$

$$(-\cos)' = \sin$$

$$F' = f \quad \Rightarrow \quad \frac{d}{dx}\left[\frac{F(ax+b)}{a}\right] = f(ax+b)$$

IOU: Fund. Th'm of Calc gives an answer.

$$\frac{d}{dx}\left[\quad ?? \quad\right] = e^{x^2}$$

There is NO "elementary" antiderivative.

WARNING: No similar formula for $f(ax^2+bx+c)$.

EXAMPLE: Find the set of all antiderivatives of $(4x+7)^{-1}$.

$$(\ln(|\bullet|))' = (\bullet)^{-1}$$

$$\frac{\ln(|4x+7|)}{4} + C \blacksquare$$

sloppy

SKILL
all antiderivs

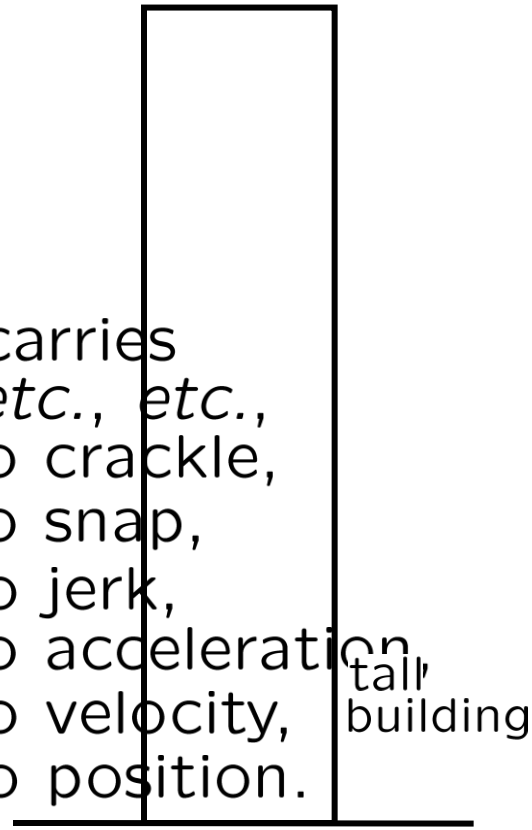
Overdot denotes d/dt.

MOTION ALONG A LINE

velocity := (position)$^\bullet$
acceleration := (velocity)$^\bullet$
jerk := (acceleration)$^\bullet$
snap := (jerk)$^\bullet$
crackle := (snap)$^\bullet$
pop := (crackle)$^\bullet$
etc., *etc.*, *etc.*

Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.

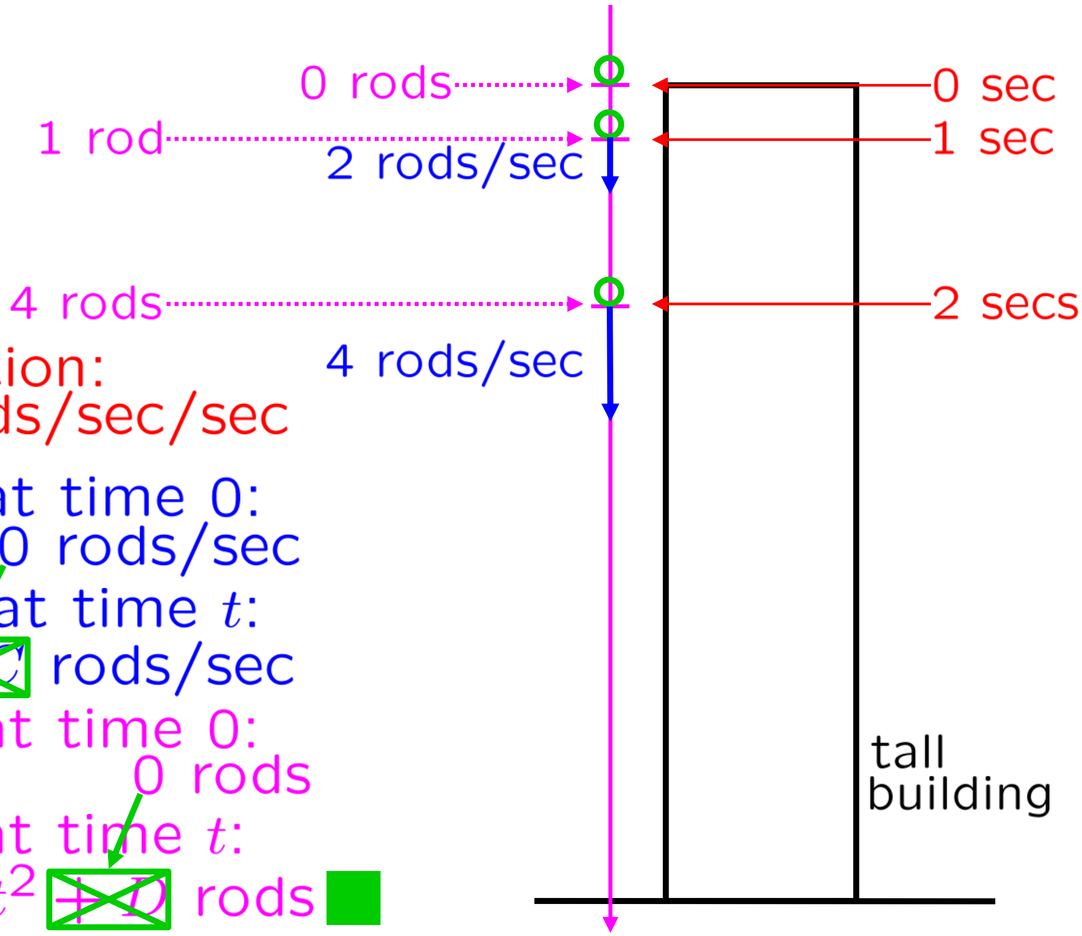
Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.



Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.

cf. EXAMPLE 3
§2.1, pp. 85-86

Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.



acceleration:
2 rods/sec/sec

velocity at time 0:
0 rods/sec
velocity at time t:
$2t$ ~~$+ C$~~ rods/sec
position at time 0:
0 rods
position at time t:
t^2 ~~$+ D$~~ rods ■

cf. EXAMPLE 3
§2.1, pp. 85-86

§7.1

Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.

EXAMPLE: A particle moving along a line has acceleration $a(t) = 5t - 4$ cm/s^2. Its initial velocity is $v(0) = -7$ cm/s and its initial position is $s(0) = 3$ cm.

Find its position function $s(t)$.

$$a(t) = 5t - 4$$

ANTIDIFF

$$v(t) = \frac{5}{2}t^2 - 4t - 7$$

ANTIDIFF

$$s(t) = \frac{5}{6}t^3 - 2t^2 - 7t + 3$$ ■

SKILL
motion on line

Integration carries
etc., *etc.*, *etc.*,
pop to crackle,
crackle to snap,
snap to jerk,
jerk to acceleration,
acceleration to velocity,
velocity to position.

STOP

EXAMPLE: A ball is thrown upward with a speed of 128 ft/s from the edge of a roof 320 ft above the ground. Find its height above the ground t seconds later. When does it hit the ground? 10 seconds after being thrown

SKILL
motion on line

$s(t)$ = height (in ft) above ground at t secs

$$s''(t) = -32$$
$$s'(t) = -32t + 128$$
$$s(t) = -16t^2 + 128t + 320$$

t_0 := time when hits ground

$$0 = -16t_0^2 + 128t_0 + 320 = -16(t_0^2 - 8t_0 - 20)$$
$$= -16(t_0 - 10)(t_0 + 2)$$