CALCULUS
Properties of the definite integral

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

not necessary

Assuming $\boxed{a < b}$ and f is contin. on $[a, b]$.

Proof: Let $F(\overset{b}{\underset{a}{x}}) := \int_a^{x^{b}_{a}} f(t)\,dt$. Then $\frac{d}{dx}[F(x)] = f(x)$.

ANTIDIFF

$c(F(x))$

Then $\int_a^b \boxed{c(f(x))}\,dx = [c(F(x))]_{x:\to a}^{x:\to b}$

$$= [c(F(b))] - [c(F(a))]$$

$$= \left[c\int_a^b f(\overset{x}{t})\,d\overset{x}{t}\right] - \cancel{\left[c\int_a^a f(t)\,dt\right]}$$

$$= c\int_a^b f(x)\,dx.$$ QED

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cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is continuous on $[a, b]$,

then $\frac{d}{dx}\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a, b)$.

§7.3

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

Similar proof.

$\int_a^b$ is linear.

$$\int_a^b [c_1(f_1(x)) + \cdots + c_n(f_n(x))]\,dx = c_1\left[\int_a^b f_1(x)\,dx\right] + \cdots + c_n\left[\int_a^b f_n(x)\,dx\right]$$

$$\int_a^b [c_1 f_1 + \cdots + c_n f_n] = c_1\left[\int_a^b f_1\right] + \cdots + c_n\left[\int_a^b f_n\right]$$

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cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

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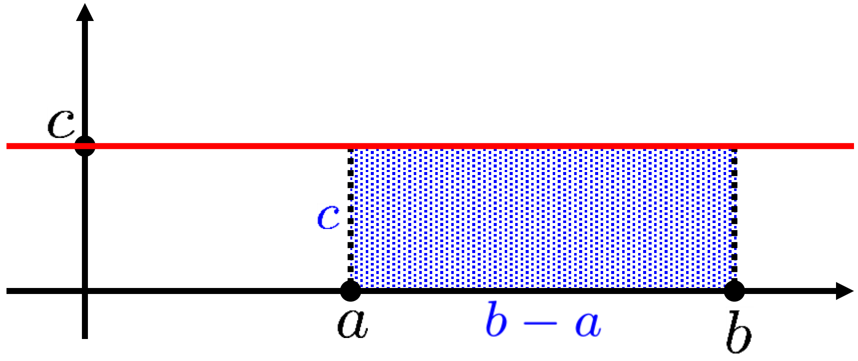
$$[cx]^{x:\to b}_{x:\to a} = c(b-a) \quad \text{QED}$$

$$\int_a^b \overset{cx}{c\,dx} = \underline{c(b-a)}$$

Suppose $c > 0$
and $a < b$.

If not . . .



cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is continuous on $[a,b]$,

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cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c\,dx = \int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int^b f(x)\,dx$$

$$\int^b$$

$$\int_a^b (f(x)) - (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$$

because subtraction is a linear combination, with coefficients $+1$ and -1.

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cf. §7.2, p. 146 THE FUNDAMENTAL THEOREM OF CALCULUS, THEOREM 7.4

If f is continuous on $[a,b]$,

then $\frac{d}{dx}\int_a^x f(t)\,dt = [f(t)]_{t:\to x} = f(x)$, for $x \in (a,b)$.

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cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) - (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$$

$$q \geq 0 \text{ on } [a,b] \qquad \Rightarrow \qquad \int_a^b q(x)\,dx \geq 0$$

Proof: $\forall$ integers $n \geq 1$, $\qquad h_n := \dfrac{b-a}{n}$

$$R_n S_a^b q = \sum_{j=1}^{n} h_n(q(a+jh_n)) \geq 0.$$

$$\int_a^b q(x)\,dx = \lim_{n\to\infty} R_n S_a^b q \geq 0.$$ QED

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) - (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$$

$q \geq 0$ on $[a,b]$ $\Rightarrow$ $\int_a^b q(x)\,dx \geq 0$

$f \geq g$ on $[a,b]$ $\Rightarrow$ $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

Proof: $q := f - g \geq 0$ on $[a,b]$, so

$$\left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right) = \int_a^b q(x)\,dx \geq 0.$$

QED

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) - (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$$

$$g \le f \text{ on } [a,b] \quad \underset{\text{monotonicity}}{\Rightarrow} \quad \int_a^b g(x)\,dx \le \int_a^b f(x)\,dx$$

$$f \ge g \text{ on } [a,b] \quad \underset{\text{monotonicity}}{\Rightarrow} \quad \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$$

$$m \le f \le M \text{ on } [a,b]$$

$$\Rightarrow \quad m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^b (f(x)) - (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) - \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^c f(x)\,dx = \left[\int_a^b f(x)\,dx\right] + \left[\int_b^c f(x)\,dx\right]$$

$f \geq g$ on $[a,b]$ $\Rightarrow$ $\displaystyle\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

$m \leq f \leq M$ on $[a,b]$

$$\Rightarrow \quad m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

the “cocycle” identity

$$\boxed{\int_a^c f(x)\,dx = \left[\int_a^b f(x)\,dx\right] + \left[\int_b^c f(x)\,dx\right]}$$

$\Nearrow F'(x) = f(x)$

cocycle identity:

$$[F(x)]_{x:\to a}^{x:\to c} = ([F(x)]_{x:\to a}^{x:\to b}) + ([F(x)]_{x:\to b}^{x:\to c})$$

cf. §7.3, p. 150ff PROPERTIES OF THE DEF. INTEGRAL

$$\int_a^b (f(x)) + (g(x))\,dx = \left(\int_a^b f(x)\,dx\right) + \left(\int_a^b g(x)\,dx\right)$$

$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$\int_a^b$ is linear.

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the "cocycle" identity

$$\int_a^c f(x)\,dx = \left[\int_a^b f(x)\,dx\right] + \left[\int_b^c f(x)\,dx\right]$$

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$$\int_a^b c(f(x))\,dx = c\int_a^b f(x)\,dx$$

$$\boxed{\int_a^b \text{ is linear.}}$$

Linear, but not multiplicative: $\dfrac{d}{dx}$, $\sum$, $\int$, $\int_a^b$, $[\bullet]_{x:\to a}^{x:\to b}$, $\triangle$

$$\frac{d}{dx}([f(x)][g(x)]) \neq \left[\frac{d}{dx}(f(x))\right]\left[\frac{d}{dx}(g(x))\right],$$

product rule (differentiation by parts)

$$\sum a_j b_j \neq \left[\sum a_j\right]\left[\sum b_j\right],$$

summation by parts

$$\int [f(x)][g(x)]\,dx \neq \left[\int f(x)\,dx\right]\left[\int g(x)\,dx\right],$$

integration by parts

$$\int_a^b [f(x)][g(x)]\,dx \neq \left[\int_a^b f(x)\,dx\right]\left[\int_a^b g(x)\,dx\right]$$

integration by parts

$$[(f(x))(g(x))]_{x:\to a}^{x:\to b} \neq ([f(x)]_{x:\to a}^{x:\to b})([g(x)]_{x:\to a}^{x:\to b})$$

product rule (eval. by parts)

$$\triangle[a_n b_n] \neq (\triangle a_n)(\triangle b_n)$$

product rule (differencing by parts)

Linear, and multiplicative: lim, $[\bullet]_{x:\to a}$

$$\lim\,[f(x)][g(x)] = [\lim\, f(x)][\lim\, g(x)]$$

$$[((f(x)(g(x))]_{x:\to a} = ([f(x)]_{x:\to a})([g(x)]_{x:\to a})$$

Linear, but not multiplicative: $\frac{d}{dx}$, $\sum$, $\int$, $\int_a^b$, $[\bullet]_{x:\to a}^{x:\to b}$, $\triangle$

$$\frac{d}{dx}([f(x)][g(x)]) \neq \left[\frac{d}{dx}(f(x))\right]\left[\frac{d}{dx}(g(x))\right],$$

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$$\int [f(x)][g(x)]\,dx \neq \left[\int f(x)\,dx\right]\left[\int g(x)\,dx\right],$$

integration by parts

$$\int_a^b [f(x)][g(x)]\,dx \neq \left[\int_a^b f(x)\,dx\right]\left[\int_a^b g(x)\,dx\right]$$

integration by parts

$$[(f(x))(g(x))]_{x:\to a}^{x:\to b} \neq ([f(x)]_{x:\to a}^{x:\to b})([g(x)]_{x:\to a}^{x:\to b})$$

product rule (eval. by parts)

$$\triangle[a_n b_n] \neq (\triangle a_n)(\triangle b_n)$$

product rule (differencing by parts)

EXAMPLE: Use the properties of the integral to evaluate $\int_0^4 (5-8x^2)\,dx$.

$$\int_0^4 (5-8x^2)\,dx = 5\left(\int_0^4 1\,dx\right) - 8\boxed{\left(\int_0^4 x^2\,dx\right)} \qquad \overset{\text{NOT}}{\left(\int_0^4 x\,dx\right)^2}$$

$\int_a^b$ is linear,

but not multiplicative . . .

EXAMPLE: Use the properties of the integral to evaluate $\int_0^4 (5-8x^2)\,dx$.

$$\int_0^4 (5-8x^2)\,dx = 5\left(\int_0^4 1\,dx\right) - 8\left(\int_0^4 x^2\,dx\right)$$

$$= 5\left([x]_{x:\to 0}^{x:\to 4}\right) - 8\left(\boxed{\left[\frac{x^3}{3}\right]_{x:\to 0}^{x:\to 4}}\right)$$

$$= 5\left([x]_{x:\to 0}^{x:\to 4}\right) - 8\left(\boxed{\frac{\boxed{[x^3]_{x:\to 0}^{x:\to 4}}}{3}}\right)$$

NOT

$$\left([x]_{x:\to 0}^{x:\to 4}\right)^3$$

$$= 5\,(4-0) - 8\left(\frac{4^3-0^3}{3}\right) = -\frac{452}{3}$$ 🟩

SKILL

Definite integration

$[\bullet]_{x:\to a}^{x:\to b}$ is also linear,

but not multiplicative . . .

EXAMPLE: Assume $\int_2^7 f(x)\,dx = 9$ and $\int_4^7 f(x)\,dx = 12.$

Compute $\int_4^2 f(x)\,dx.$

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the "cocycle" identity

$$\left(\int_2^4 f(x)\,dx\right) + \underbrace{\left(\int_4^7 f(x)\,dx\right)}_{12} = \underbrace{\int_2^7 f(x)\,dx}_{9}$$

$$\int_2^4 f(x)\,dx = 9 - 12 = -3$$

SKILL

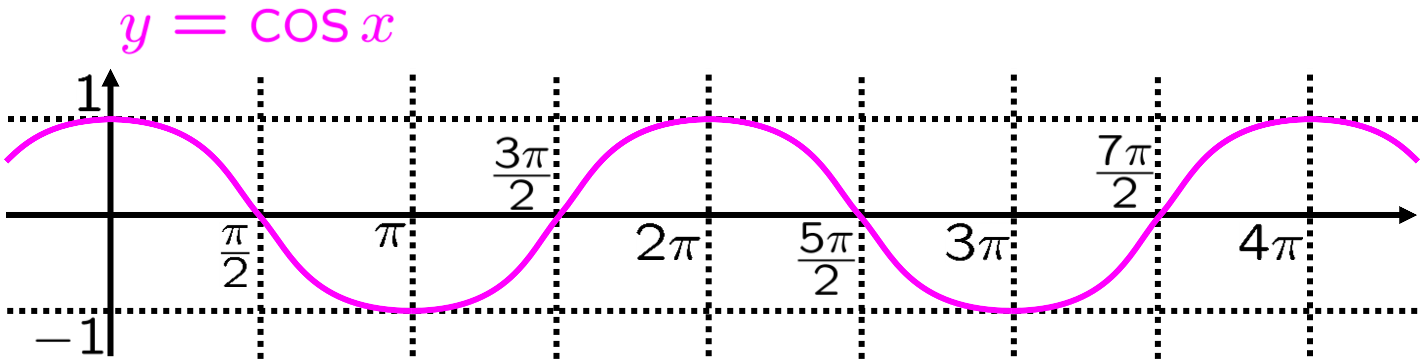
Properties of integration

$$\int_4^2 f(x)\,dx = -\int_2^4 f(x)\,dx = -(-3) = 3$$ ■

$$\boxed{\int_b^a f(x)\,dx} := -\int_a^b f(x)\,dx, \quad \text{if } a < b$$

EXAMPLE: Compute $\int_0^{7\pi/2} |\cos x|\,dx$.

Where positive & negative on $[0, 7\pi/2]$?



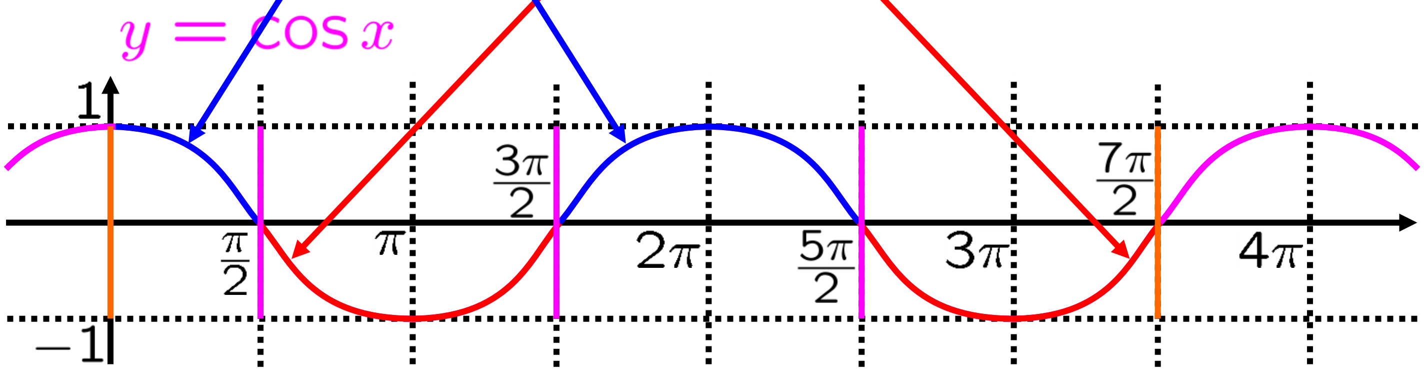
EXAMPLE: Compute $\int_0^{7\pi/2} |\cos x|\, dx$.

$$\Vert$$

COCYCLE IDENTITY

$$\left[\int_0^{\pi/2} |\cos x|\, dx\right] + \left[\int_{\pi/2}^{3\pi/2} |\cos x|\, dx\right] + \left[\int_{3\pi/2}^{5\pi/2} |\cos x|\, dx\right] + \left[\int_{5\pi/2}^{7\pi/2} |\cos x|\, dx\right]$$

Where positive & negative on $[0, 7\pi/2]$?



EXAMPLE: Compute $\int_0^{7\pi/2} |\cos x|\,dx$.

COCYCLE IDENTITY

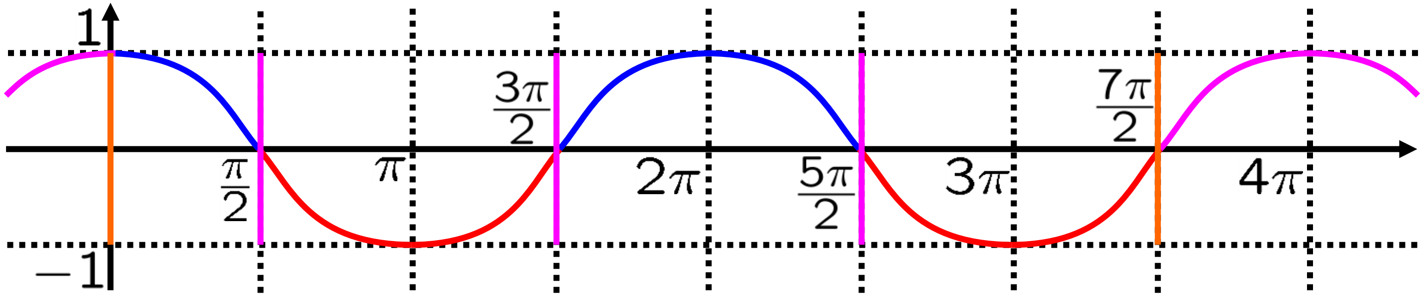
$$\| $$

$$\left[\int_0^{\pi/2} |\cos x|\,dx\right] + \left[\int_{\pi/2}^{3\pi/2} |\cos x|\,dx\right] + \left[\int_{3\pi/2}^{5\pi/2} |\cos x|\,dx\right] + \left[\int_{5\pi/2}^{7\pi/2} |\cos x|\,dx\right]$$

$$\| $$

$$\left[\int_0^{\pi/2} (\cos x)\,dx\right] + \left[\int_{\pi/2}^{3\pi/2} (-\cos x)\,dx\right] + \left[\int_{3\pi/2}^{5\pi/2} (\cos x)\,dx\right] + \left[\int_{5\pi/2}^{7\pi/2} (-\cos x)\,dx\right]$$

$y = \cos x$



§8.1

EXAMPLE: Compute $\int_0^{7\pi/2} |\cos x|\, dx$.

$$\|$$

$$\left[\int_0^{\pi/2} (\cos x)\, dx\right] + \left[\int_{\pi/2}^{3\pi/2} (-\cos x)\, dx\right] +$$

$$\left[\int_{3\pi/2}^{5\pi/2} (\cos x)\, dx\right] + \left[\int_{5\pi/2}^{7\pi/2} (-\cos x)\, dx\right]$$

$$\left[\int_0 [\sin x]_{x:\to 0}^{x:\to \pi/2} \quad x\right] + \left[\int_{\pi/2}^{3\pi/2} (-\cos x)\, dx\right] +$$

$$\left[\int_{3\pi/2}^{5\pi/2} (\cos x)\, dx\right] + \left[\int_{5\pi/2}^{7\pi/2} (-\cos x)\, dx\right]$$

EXAMPLE: Compute $\displaystyle\int_0^{7\pi/2} |\cos x|\,dx$.

$$\|$$

$$\left[\int_0^{\pi/2} (\cos x)\,dx\right] + \left[\int_{\pi/2}^{3\pi/2} (-\cos x)\,dx\right] + \left[\int_{3\pi/2}^{5\pi/2} (\cos x)\,dx\right] + \left[\int_{5\pi/2}^{7\pi/2} (-\cos x)\,dx\right]$$

$$\|$$

$$[\underset{0}{\overset{1}{\sin}}\, x]_{x:\to 0}^{x:\to \pi/2} \quad + \quad [-\underset{-1}{\overset{-(-1)}{\sin}}\, x]_{x:\to \pi/2}^{x:\to 3\pi/2} \quad + \quad [\underset{-1}{\overset{1}{\sin}}\, x]_{x:\to 3\pi/2}^{x:\to 5\pi/2} \quad + \quad [-\underset{-1}{\overset{-(-1)}{\sin}}\, x]_{x:\to 5\pi/2}^{x:\to 7\pi/2}$$

$$\|$$

$$[1 - 0] \quad + \quad [-(-1) - (-1)] \quad + \quad [1 - (-1)] \quad + \quad [-(-1) - (-1)]$$

$$\|$$

7 🟩

SKILL
find def int

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

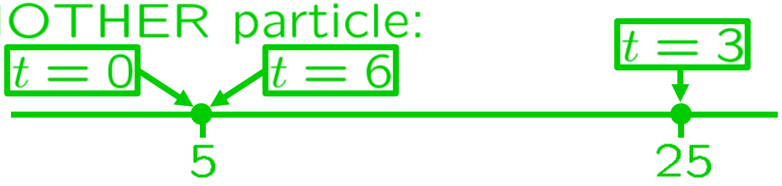
(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $\displaystyle\int_1^4 v(t)\,dt$

(b) $\displaystyle\int_1^4 \boxed{|v(t)|}\,dt$

speed

ANOTHER particle:



displacement during $1 \leq t \leq 6$: 0

distance traveled during $1 \leq t \leq 6$: 40

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $$\int_1^4 v(t)\,dt = \int_1^4 t^2 + 2t - 15\,dt$$

$$= \left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 1}^{t:\to 4}$$

LINEARITY OF $[\bullet]_{x:\to a}^{x:\to b}$

$$= \left(\frac{4^3 - 1^3}{3}\right) + (4^2 - 1^2) - 15(4 - 1)$$

$$= \tfrac{63}{3} + (16 - 1) - 15(3)$$

$$= 21 + 15 - 45$$

$$= 36 - 45$$

$$= -9$$

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $\int_1^4 v(t)\,dt = -9$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

$= -9$

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

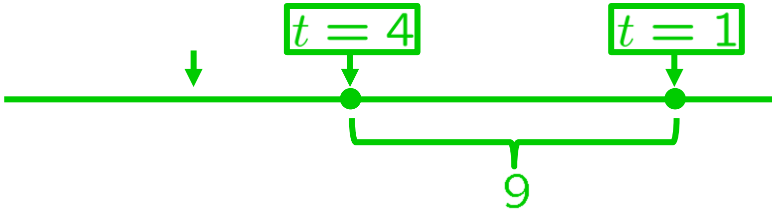
(a) Find the displacement of the particle during the time period $1 \le t \le 4$.

(b) Find the distance traveled during the time period $1 \le t \le 4$.

(a) $\int_1^4 v(t)\,dt = -9$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

(b) $\int_1^4 |v(t)|\,dt$

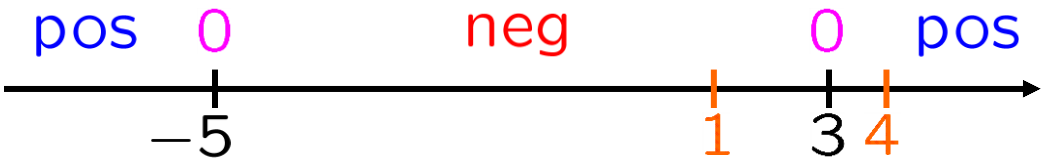


It'll turn out that the distance traveled is > 9.

$v(t) = (t - 3)(t + 5)$

moving left
$t = 1$ to $t = 3$

moving right
$t = 3$ to $t = 4$



EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $\displaystyle\int_1^4 v(t)\,dt = -9$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

(b)
$$\int_1^4 |v(t)|\,dt = \left[\int_1^3 |v(t)|\,dt\right] + \left[\int_3^4 |v(t)|\,dt\right]$$

COCYCLE IDENTITY

$$= \left[\int_1^3 -(v(t))\,dt\right] + \left[\int_3^4 (v(t))\,dt\right]$$

$$= \left[\int_1^3 -(t^2 + 2t - 15)\,dt\right] + \left[\int_3^4 (t^2 + 2t - 15)\,dt\right]$$

moving left $t = 1$ to $t = 3$

moving right $t = 3$ to $t = 4$

$v(t) = (t-3)(t+5)$

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $\displaystyle\int_1^4 v(t)\,dt = -9$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

(b)
$$\int_1^4 |v(t)|\,dt = \left[\int_1^3 |v(t)|\,dt\right] + \left[\int_3^4 |v(t)|\,dt\right]$$
$$= \left[\int_1^3 -(v(t))\,dt\right] + \left[\int_3^4 (v(t))\,dt\right]$$
$$= \left[\int_1^3 -(t^2 + 2t - 15)\,dt\right] + \left[\int_3^4 (t^2 + 2t - 15)\,dt\right]$$
$$= -\left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 1}^{t:\to 3} + \left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 3}^{t:\to 4}$$

§8.1 LINEARITY

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \le t \le 4$.

(b) Find the distance traveled during the time period $1 \le t \le 4$.

(a) $\int_1^4 v(t)\,dt = -9$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

(b) $$\int_1^4 |v(t)|\,dt = -\left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 1}^{t:\to 3} + \left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 3}^{t:\to 4}$$

$$= -\left[\left(\frac{3^3}{3} + 3^2 - 15 \cdot 3\right) - \left(\frac{1^3}{3} + 1^2 - 15 \cdot 1\right)\right]$$

$$+\left[\left(\frac{4^3}{3} + 4^2 - 15 \cdot 4\right) - \left(\frac{3^3}{3} + 3^2 - 15 \cdot 3\right)\right]$$

$$= -\left[\frac{}{3} + t^2 - 15t\right]_{t:\to 1} + \left[\frac{}{3} + t^2 - 15t\right]_{t:\to 3}$$

EXAMPLE: A particle moves along a line so that its velocity at time t is $v(t) = t^2 + 2t - 15$ (measured in miles per hour).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during the time period $1 \leq t \leq 4$.

(a) $$\int_1^4 v(t)\,dt = -9$$

Position at $t = 4$ is 9 units to the left of position at $t = 1$.

(b) $$\int_1^4 |v(t)|\,dt = -\left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 1}^{t:\to 3} + \left[\frac{t^3}{3} + t^2 - 15t\right]_{t:\to 3}^{t:\to 4}$$

$$= -\left[\left(\frac{3^3}{3} + 3^2 - 15 \cdot 3\right) - \left(\frac{1^3}{3} + 1^2 - 15 \cdot 1\right)\right]$$

$$+\left[\left(\frac{4^3}{3} + 4^2 - 15 \cdot 4\right) - \left(\frac{3^3}{3} + 3^2 - 15 \cdot 3\right)\right]$$

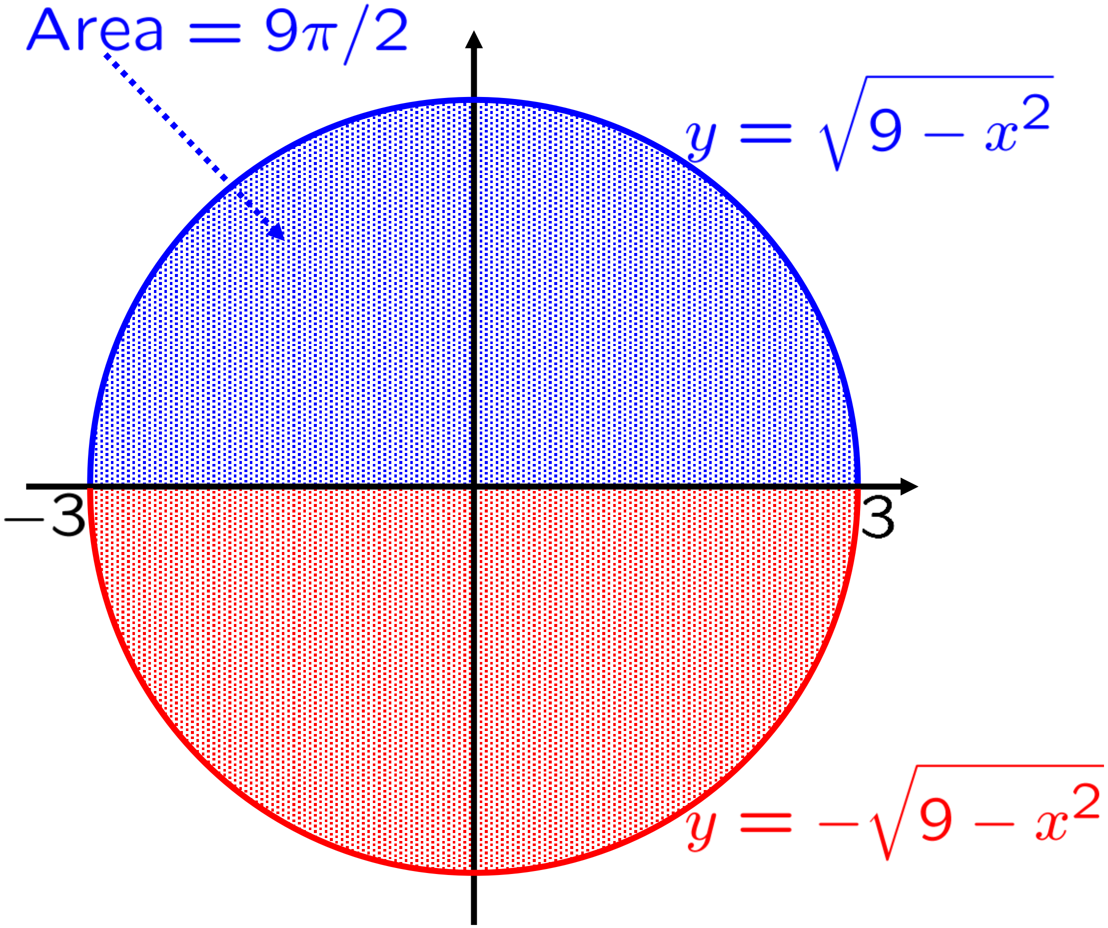
SKILL
compute displacement
and distance traveled

$$= \overset{\substack{\text{moving left} \\ t = 1 \text{ to } t = 3}}{-\left[-\frac{40}{3}\right]} + \overset{\substack{\text{moving right} \\ t = 3 \text{ to } t = 4}}{\left[\frac{13}{3}\right]} = \frac{53}{3} \doteq 17.667$$ 🟩

EXAMPLE:

(a) Compute $\int_{-3}^{3} -\sqrt{9-x^2}\,dx$.

(b) Compute $\int_{3}^{-3} -\sqrt{9-x^2}\,dx$.



$$\int_{-3}^{3} \sqrt{9-x^2}\,dx = 9\pi/2$$

$$\int_{-3}^{3} -\sqrt{9-x^2}\,dx = -9\pi/2$$

EXAMPLE:

(a)Compute $\int_{-3}^{3} -\sqrt{9-x^2}\,dx$. (b)Compute $\int_{3}^{-3} -\sqrt{9-x^2}\,dx$.

(a) $\int_{-3}^{3} -\sqrt{9-x^2}\,dx = -9\pi/2$

$$\int_{-3}^{3} -\sqrt{9-x^2}\,dx = -9\pi/2$$

EXAMPLE:

(a) Compute $\int_{-3}^{3} -\sqrt{9-x^2}\,dx$.

(b) Compute $\int_{3}^{-3} -\sqrt{9-x^2}\,dx$.

$$\text{(a)} \int_{-3}^{3} -\sqrt{9-x^2}\,dx = -9\pi/2$$

SKILL
Properties of integration

$$\text{(b)} \int_{3}^{-3} -\sqrt{9-x^2}\,dx = -\int_{-3}^{3} -\sqrt{9-x^2}\,dx = 9\pi/2 \;\blacksquare$$

$$\boxed{\int_b^a f(x)\,dx} := -\int_a^b f(x)\,dx, \quad \text{if } a < b$$

SKILL

Properties of integration

Whitman problems
§7.3, p. 154, #1-6

STOP