CALCULUS
Volume by cylindrical shells
FORM A VERTICAL LINE SEGMENT IN THE PLANE.
REVOLVE ABOUT THE VERTICAL AXIS.
Form a vertical line segment in the plane.

Revolve about the vertical axis.

Area of the resulting "shell"?
FORM A VERTICAL LINE SEGMENT IN THE PLANE.

REVOLVE ABOUT THE VERTICAL AXIS.

AREA OF THE RESULTING "SHELL"?

\[ \text{circumference} = 2\pi(3) \]
\[ \text{radius} = 3 \]

\[ 6 \]

\[ 3 \]
FORM A VERTICAL LINE SEGMENT IN THE PLANE.

REVOLVE ABOUT THE VERTICAL AXIS.

AREA OF THE RESULTING "SHELL"?

circumference $= 2\pi(3)$

height $= 6$

CUT HERE AND OPEN.

§9.3
FORM A VERTICAL LINE SEGMENT IN THE PLANE.

REVOLVE ABOUT THE VERTICAL AXIS.

AREA OF THE RESULTING “SHELL”?

ANSWER: 

\[ 2\pi(3)[6] \]

\[ 2\pi(3) \]

CUT HERE AND OPEN.

\[ \text{AREA OF SHELL} = [\text{circumference}][\text{height}] \]

\[ \text{§9.3} \]
EXAMPLE: Find the volume of the solid formed by revolving, about the \( y \)-axis, the region bounded by \( y = 8x^2 - x^5 \) and \( y = 0 \).

Easier solution... \[
\text{volume} = \int_0^? \left( \pi[??^2] - \pi[?^2] \right) \, dy
\]

HARD ???

\[y = 8x^2 - x^5\]

\[\text{area} = \pi[??^2] - \pi[?^2]\]

WASHER

\[?? \quad \text{expr. of } y \quad ??\]

\[\text{WAY HARD}\]

\(9.3\)
EXAMPLE: Find the volume of the solid formed by revolving, about the $y$-axis, the region bounded by $y = 8x^2 - x^5$ and $y = 0$.

Easier solution...
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Easier solution...

\[ \text{AREA OF SHELL} = [\text{circumference}][\text{height}] \]

area of shell $= [2\pi x][8x^2 - x^5]$
EXAMPLE: Find the volume of the solid formed by revolving, about the y-axis, the region bounded by $y = 8x^2 - x^5$ and $y = 0$.

Easier solution...

$$\text{volume} = \int_0^2 \left( [2\pi x] [8x^2 - x^5] \right) dx = \cdots$$

$$\text{area of shell} = [2\pi x][8x^2 - x^5]$$

$y = 8x^2 - x^5$

**SKILL**

shell method

Next: General formulas...
Say \( f \geq 0 \) on \([a, b]\). The volume of the solid formed by revolving, about the \( y \)-axis, the region under the curve

\[ y = f(x) \text{ from } x = a \text{ to } x = b, \text{ is } \int_a^b [2\pi x][f(x)] \, dx. \]

**Area of Shell** = \([2\pi x][f(x)]\)

**Area of Shell** = \([\text{circumference}][\text{height}]\)
VARIATION: Suppose \( f(x) \leq g(x) \), for \( a \leq x \leq b \). The volume of the solid formed by revolving, about the \( y \)-axis, the region between \( y = f(x) \) and \( y = g(x) \) from \( x = a \) to \( x = b \), is

\[
\int_{a}^{b} \left[ 2\pi x \right] \left[ (g(x)) - (f(x)) \right] \, dx.
\]

\[ \text{area of shell} = \left[ 2\pi x \right] \left[ (g(x)) - (f(x)) \right] \]

\[ \text{shell} \]

\[ y = g(x) \]

\[ y = f(x) \]

\[ a \]

\[ b \]

\[ \text{AREA OF SHELL} = \text{[circumference][height]} \]

\[ \text{§9.3} \]
Say $c < a$. **VARIATION:** Suppose $f(x) \leq g(x)$, for $a \leq x \leq b$. The volume of the solid formed by revolving, about the $x = c$, the region between $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$, is

$$\int_a^b [2\pi(x - c)][(g(x)) - (f(x))] \, dx$$

**area of shell** = $[2\pi(x - c)][(g(x)) - (f(x))]$

**Exercise:**

$c > b$

Next: $x$ as a function of $y$...

**AREA OF SHELL** = [circumference][height]
Say \( f \geq 0 \) on \([a, b]\). The vol. of the solid formed by revolving, about the \( x \)-axis, the region between the \( y \)-axis and the curve \( x = f(y) \) from \( y = a \) to \( y = b \), is \( \int_a^b [2\pi y][f(y)] \, dy \).
VARIATION: Suppose \( f(y) \leq g(y) \), for \( a \leq y \leq b \). The volume of the solid formed by revolving, about the \( x \)-axis, the region between \( x = f(y) \) and \( x = g(y) \), from \( y = a \) to \( y = b \), is \[ \int_a^b [2\pi y][(g(y)) - (f(y))] \, dy. \]

\[
\text{area of shell} = \int_a^b [2\pi y][(g(y)) - (f(y))] \, dy.
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Say $c < a$. VARIATION: Suppose $f(y) \leq g(y)$, for $a \leq y \leq b$. The volume of the solid formed by revolving, about the $y = c$, the region between $x = f(y)$ and $x = g(y)$ from $y = a$ to $y = b$, is

$$
\int_a^b [2\pi(y - c)][(g(y)) - (f(y))] \, dy.
$$

area of shell $= [2\pi(y - c)][(g(y)) - (f(y))]$