MATH 4604 Spring 2017, Final exam
Handout date: Thursday 11 May 2017
Instructor: Scot Adams

PRINT YOUR NAME:

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Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let V and W be finite dimensional vector spaces, and let $f : V \dashrightarrow W$. Let $p \in V$. Then $\mathrm{LINS}_p(f) = \cdots$

B. (5 pts) Let V and W be vector spaces, $f : V \dashrightarrow W$. Let $x \in \mathrm{dom}[f]$. Let $v \in V$. Then $SS^f_{x,v} : DSS^f_{x,v} \to W$ is defined by $SS^f_{x,v}(h) = \cdots$

C. (5 pts) Let V and W be finite dimensional vector spaces and let $p \geqslant 0$. Then $\mathcal{O}_{V,W}(p) := \cdots$

D. (5 pts) Let $n \in \mathbb{N}$ and let $h : \mathbb{R}^n \dashrightarrow \mathbb{R}^n$. Then h is **near constant** (or **half-Lipschitz**) means . . .

E. (5 pts) Let $n \in \mathbb{N}$ and let $A \in \mathbb{R}^{n \times n}_{\text{sym}}$. Then A **is positive definite** means: . . .

F. (5 pts) Let $d \in \mathbb{N}$. Let $S \subseteq \mathbb{R}^d$ be bounded. Then $\overline{\mu}_d(S) := \cdots$

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R}^5 \dashrightarrow \mathbb{R}^8$. Assume, for all $i \in \{1, \ldots, 5\}$, that $\partial_i f \in \mathcal{O}_{5,8}(1)$. Then $f \in \mathcal{O}_{5,8}(2)$.

b. (5 pts) $\mathcal{O} \circ \mathcal{O} \subseteq \mathcal{O}$.

c. (5 pts) Let V be a finite dimensional vector space and let $n := \dim V$. Then V is isomorphic to $\mathbb{R}^n$.

d. (5 pts) Let V be a fdVS. Let $|\bullet|, \|\bullet\| \in \mathcal{N}(V)$. Then $|\bullet| \approx \|\bullet\|$.

e. (5 pts) Let $\mathcal{S}$ be a semiring of sets. Then $\langle \mathcal{S} \rangle_{\mathrm{fin}\amalg}$ is a ring of sets.

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I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Let V and W both be finite dimensional vector spaces. Let $P \in \mathcal{P}_4(V, W)$. Let $F \in \mathrm{SM}_4(V, W)$ denote the polarization of P. Let $x, u \in V$. Find a formula for $(D_x P)(u)$, in terms of F, x and u.

2. (10 pts) Find a function $f \in \mathcal{O}(1)$ such that $0 \notin \mathrm{dom}[f']$.

3. (10 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume $f \in \mathcal{O}(1)$. Show: $0 \in \mathrm{dom}[f']$.

4. (15 pts.) Let $\| \bullet \| := | \bullet |_2 \in \mathcal{N}(\mathbb{R}^2)$. Let $f : \mathbb{R}^2 \dashrightarrow \mathbb{R}$. Assume $(0,0) \in \text{dom}[f]$ and $f(0,0) = 0$. Define $P \in \mathcal{Q}(\mathbb{R}^2, \mathbb{R})$ by $P(x,y) = xy$. Assume $f - P \in \mathcal{O}_{2,1}(2)$. Show f does not have a local semi-minimum at $(0,0)$. That is, show, for all $\delta > 0$, that there exists $z \in \mathbb{R}^2$ such that: both ($\|z\| < \delta$) and ($f(z) < 0$).