MATH 4604 Spring 2017, Midterm #1
Handout date: Thursday 23 February 2017
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR @umn.edu EMAIL ADDRESS:

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let X be a topological space, $f : X \dashrightarrow \mathbb{R}$ and $a \in \text{dom}[f]$. Then f **has a local strict max near** a **in** X means ...

B. (5 pts) (Fill in **BOTH** "..."s in the following.) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Let $a \in \text{dom}[f]$. Then $SS_a^f : \cdots \to \mathbb{R}$ is defined by ...

C. (5 pts) We defined $\mathcal{O} := \cdots$.

D. (5 pts) We defined $\mathcal{O}(3) := \cdots$.

E. (5 pts) Let V and W be vector spaces. Then $SBF(V, W) := \cdots$.

F. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $S \subseteq \mathbb{R}$. Then f is **differentiable on** S means ...

II. True or false (no partial credit):

a. (5 pts) Let V be a vector space, $L, M \in \mathcal{L}(V)$. Then $LM \in \mathcal{Q}(V)$.

b. (5 pts) $\mathcal{O}(1) \subseteq o(1)$.

c. (5 pts) If V and W are isomorphic vector spaces and V is finite dimensional, then W is also finite dimensional.

d. (5 pts) Let V be a finite dimensional vector space and let $n := \dim V$. Then V is isomorphic to $\mathbb{R}^n$.

e. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$. Assume f is differentiable on $\mathbb{R}$ and f is semi-increasing on $\mathbb{R}$. Then, for all $x \in \mathbb{R}$, $f'(x) \geqslant 0$.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Give an example of a polynomial $f : \mathbb{R} \to \mathbb{R}$ such that
   - f is strictly increasing on $\mathbb{R}$ and
   - $f'(0) = f'(3) = 0$.

2. (10 pts) Let $f : \mathbb{R} \to \mathbb{R}$, and assume that f is differentiable on $\mathbb{R}$. Assume that $f(0) = 0$ and that $f' \in \mathcal{O}$. Show that $f \in \mathcal{O}(1)$.

3. (10 pts) Let $f \in \mathcal{O}(2)$ and let $g \in \mathcal{O}(3)$. Show that $g \circ f \in \mathcal{O}(6)$.

NOTE: You may use, without proof, that $\mathcal{O} \circ o \subseteq \mathcal{O}$, that $|\mathcal{O}| \subseteq \mathcal{O}$ and that $\mathcal{O} \cdot \mathcal{O} \subseteq \mathcal{O}$.

4. (15 pts.) Let $S \subseteq \mathbb{R}$. Recall that S is **bounded in** $\mathbb{R}$ means: there exist $p, q \in \mathbb{R}$ such that $p \leqslant S \leqslant q$.

Let $f_1, f_2, f_3, \ldots : \mathbb{R} \to \mathbb{R}$ be a sequence of functions. Assume that $\{f_1(0), f_2(0), f_3(0), \cdots\}$ is bounded in $\mathbb{R}$. Assume, for all $j \in \mathbb{N}$, for all $x \in \mathbb{R}$, that f_j is differentiable at x and that $|f_j'(x)| \leqslant 3$. Show that $\{f_1(1), f_2(1), f_3(1), \ldots\}$ is bounded in $\mathbb{R}$.