MATH 4604 Spring 2017, Midterm #2
Handout date: Thursday 23 March 2017
Instructor: Scot Adams

PRINT YOUR NAME:

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Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let V and W be vector spaces. Then $\mathcal{Q}(V,W) := \cdots$

B. (5 pts) Let V and W be vector spaces. Then $\mathrm{SBF}(V,W) := \cdots$

C. (5 pts) Let V be a finite dimensional vector space, $n := \dim V$, $\mathcal{B} \in \mathrm{OB}(V)$ an ordered basis. Then $\mathcal{B}$ determines a topology, $\mathcal{T}_{\mathcal{B}} := \cdots$.

D. (5 pts) Let V be a vector space and let $\| \bullet \| : V \to [0, \infty)$. Then $\| \bullet \|$ is a **norm on** V if ...

E. (5 pts) Let V be a vector space and let $B \in \mathrm{SBF}(V, \mathbb{R})$. Then B **is positive definite** means: ...

F. (5 pts) Let $n \in \mathbb{N}$ and let $A \in \mathbb{R}^{n\times n}_{\mathrm{sym}}$. Then $A \geqslant 0$ means: ...

II. True or false (no partial credit):

a. (5 pts) Let $F : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ denote the cofactor map, so, $\forall A \in \mathbb{R}^{2\times 2}$, $F(A)$ is the cofactor matrix of A. Then $F \in \mathcal{P}_3(\mathbb{R}^{2\times 2}, \mathbb{R}^{2\times 2})$.

b. (5 pts) Let V be a finite dimensional vector space. Then, for all $\mathcal{A}, \mathcal{B} \in \mathrm{OB}(V)$, we have $\mathcal{T}_{\mathcal{A}} = \mathcal{T}_{\mathcal{B}}$.

c. (5 pts) We have $[\mathcal{O}(2)] \cdot [\mathcal{O}(4)] \subseteq \mathcal{O}(8)$.

d. (5 pts) There exists a vector space isomorphism $\mathbb{R}^5 \to \mathbb{R}^6$.

e. (5 pts) Let V and W be vector spaces and let $B \in \mathcal{B}(V, V, W)$. Then there exists $S \in \mathrm{SBF}(V, W)$ such that, $\quad \forall v \in V,\ S(v, v) = B(v, v)$.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Find $f \in o$ and $g \in \mathcal{O}$ such that $g \circ f \notin o$.

2. (10 pts) Let $n \in \mathbb{N}$. Let $\cdot : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ denote the dot product. Let $A \in \mathbb{R}^{n \times n}$. Recall that $A^t \in \mathbb{R}^{n \times n}$ denotes the transpose of A. Let $v, w \in \mathbb{R}^n$. Show that $v \cdot [L_A(w)] = [L_{A^t}(v)] \cdot w$.

3. (10 pts) Let V, W and X be vector spaces. Let $B \in \mathcal{B}(V, W, X)$. Show: $B \in \mathcal{Q}(V \times W, X)$. That is, show: $\exists A \in \mathrm{SBF}(V \times W, X)$ such that $\forall v \in V$, $\forall w \in W$, $A((v, w), (v, w)) = B(v, w)$.

4. (15 pts.) Let $\psi \in \mathcal{o}(7)$ and $\phi \in \mathcal{O}(1)$. Show that $\psi \circ \phi \in \mathcal{o}(7)$.

NOTE: You may use without proof, all of the following facts:

$$|\mathcal{O}| \subseteq \mathcal{O}, \qquad \mathcal{O} \cdot \mathcal{o} \subseteq \mathcal{o}, \qquad \mathcal{o} \circ \mathcal{o} \subseteq \mathcal{o}, \qquad \mathcal{O}(1) \subseteq \mathcal{o},$$

$$\forall n \in \mathbb{N},\ \mathcal{O}^n \subseteq \mathcal{O}.$$