Homework for MATH 4604 (Advanced Calculus II)
Spring 2017

Homework 14: Due on Tuesday 2 May

55. Let $m, n \in \mathbb{N}$, $A \in \mathbb{R}^{m\times n}$ and $v \in \mathbb{R}^n$. Show: $|L_A(v)|_2 \leqslant |A|_2 \cdot |v|_2$.

56. Let $n \in \mathbb{N}$ and let $A \in \mathbb{R}^{n\times n}$. Let I_n denote the $n \times n$ identity matrix. Assume that $|I_n - A|_2 \leqslant 1/2$. Show that A is invertible.

57. Let $m, n \in \mathbb{N}$ and let $g : \mathbb{R}^m \dashrightarrow \mathbb{R}^n$. Assume $0_m \in \operatorname{dom}[Dg]$. Assume $g(0_m) = 0_n$. Assume, for all $j \in \{1, \ldots, m\}$, $(\partial_j g)(0_m) = 0_n$. Show that $g \in \mathcal{o}_{mn}(1)$.

Homework 13: Due on Tuesday 25 April

50. Let V and W be finite dimensional vector spaces. Show

(1) $\mathcal{Q}(V, W) \subseteq \mathcal{O}_{VW}(2)$, and
(2) $[\mathcal{Q}(V, W)] \cap [\mathcal{o}_{VW}(2)] = \{\mathbf{0}_{VW}\}$.

51. Let $m \in \mathbb{N}$, $q \in \mathbb{R}$, $v \in \mathbb{R}^m$, $S \in \mathbb{R}^{m\times m}_{\text{sym}}$. Define $C \in \mathcal{C}(\mathbb{R}^m, \mathbb{R})$ and $L \in \mathcal{L}(\mathbb{R}^m, \mathbb{R})$ and $Q \in \mathcal{Q}(\mathbb{R}^m, \mathbb{R})$ by $C(x) = q$ and $L(x) = (x^H v^V)_{11}$ and $Q(x) = ((x^H S x^V)_{11})/(2!)$. Let $f := C + L + Q \in \mathcal{P}_{\leqslant 2}(\mathbb{R}^m, \mathbb{R})$. Let $I := \{1, \ldots, m\}$. Show

(1) $f(0_m) = q$,
(2) $\forall i \in I,\ (\partial_i f)(0_m) = v_i$ and
(3) $\forall i, j \in I,\ (\partial_i \partial_j f)(0_m) = S_{ij}$.

52. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and $\delta > 0$. Let $I := (-\delta, \delta)$. Assume that f' is defined on I, *i.e.*, that $I \subseteq \operatorname{dom}[f']$. Show that there exists $c : I \to I$ such that $c \in \mathcal{O}(1)$ and such that, for all $x \in I$, we have $[f(x)] - [f(0)] = [f'(c(x))]x$.

53. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $g : \mathbb{R}^2 \dashrightarrow \mathbb{R}$. Assume: $\forall t \in \mathbb{R},\ f(t) = g(t, 0)$. Show: $\forall t \in \mathbb{R},\ f'(t) = (\partial_1 g)(t, 0)$.

54. Let $f : \mathbb{R}^2 \dashrightarrow \mathbb{R}$. Assume $0_2 \in \operatorname{dom}[D^2 f]$. Show $\widehat{R}^f \in \mathcal{o}_{21}(2)$.

Hint: Show: for all $x, y \in \mathbb{R}$,

$$\widehat{R}^f(x,y) = \Big[\ f(x,y)\ \Big] - \Big[\ (f(0_2)) + ((\partial_1 f)(0_2))x + ((\partial_2 f)(0_2))y + [\,1/(2!)\,]\,[\,((\partial_1\partial_1 f)(0_2))x^2 + 2((\partial_1\partial_2 f)(0_2))xy + ((\partial_2\partial_2 f)(0_2))y^2\ ]\ \Big].$$

Then show: at 0_2,

$$0 = \widehat{R}^f = \partial_1(\widehat{R}^f) = \partial_2(\widehat{R}^f) = \partial_1\partial_1(\widehat{R}^f) = \partial_1\partial_2(\widehat{R}^f) = \partial_2\partial_2(\widehat{R}^f).$$

Homework 12: Due on Tuesday 18 April

47. Let $m \in \mathbb{N}$ and let W be a finite dimensional vector space. Let $f : \mathbb{R}^m \dashrightarrow W$. Let $p \in \mathbb{R}^m$ and let $i \in \{1, \ldots, m\}$. Show:

$$(\partial_i f)(p + \bullet) \quad = \quad \partial_i(f(p + \bullet)).$$

48. Let V, W and X be finite dimensional vector spaces and let $p \in V$. Let $f : V \dashrightarrow W$, $g : V \dashrightarrow X$. Show that $D_p((f,g)) = (D_p f, D_p g)$.

49. Let $m, n \in \mathbb{N}$, let $f : \mathbb{R}^m \dashrightarrow \mathbb{R}^n$, let $k \in \mathbb{N}_0$ and let $p \in \mathbb{R}^m$. Assume, for all $i \in \{1, \ldots, m\}$, that $\partial_i f$ is both defined near p and bounded near p. Show both of the following:

(1) $f_p^T \in \mathcal{O}_{mn}(1)$ and
(2) f is continuous at p.

Hint: For (1), simply capitalize all the $\mathcal{o}$s appearing in the proof of the corresponding result about $\mathcal{o}$ proved during class. For (2), combine (1) with $\mathcal{O}_{mn}(1) \subseteq \mathcal{o}_{mn}$ to see that $f_p^T \in \mathcal{o}_{mn}$.

Homework 11: Due on Tuesday 11 April

44. Let V, W and X be vector spaces, let $f : V \dashrightarrow W$, $g : W \dashrightarrow X$ and let $p \in \mathrm{dom}\,[g \circ f]$. Show that $(g \circ f)_p^T = (g_{f(p)}^T) \circ (f_p^T)$.

Hint: Given $v \in V$. We wish to show: $(g \circ f)_p^T(v) = (g_{f(p)}^T)((f_p^T)(v))$. To keep the notation from getting messy, I suggest defining $q := f(p)$ and $w := f_p^T(v)$. Then compare $(g \circ f)_p^T(v) = [g(f(p+v))] - [g(f(p))]$ with $(g_{f(p)}^T)((f_p^T)(v)) = g_q^T(w) = [g(q+w)] - [g(q)]$.

45. Let U and V be finite dimensional vector spaces, let $f : U \dashrightarrow V$ and let $p \in \mathrm{dct}[f]$. Assume $\mathrm{dom}\,[f]$ is a nbd of p in U. Show $f_p^T \in \mathcal{o}_{UV}$.

Note: $f_p^T = [f(p + \bullet)] - [f_p^C]$.

Unassigned HW: Let U and V be finite dimensional vector spaces, let $f : U \dashrightarrow V$ and let $p \in \mathrm{dlin}[f]$. Show that $f_p^T \in \mathcal{O}_{UV}(1)$.

Note: Paige showed us, in class, how to do the Unassigned HW above.

46. Let U, V, W and X be vector spaces, let $* \in \mathcal{B}(V,W,X)$ and let $m,n \in \mathbb{N}_0$. Show: $[\mathcal{P}_m(U,V)] * [\mathcal{P}_n(U,W)] \subseteq [\mathcal{P}_{m+n}(U,X)]$.

Hint: Given $P \in \mathcal{P}_m(U,V)$ and $Q \in \mathcal{P}_n(U,W)$. We wish to show: $P * Q \in \mathcal{P}_{m+n}(U,X)$. We wish to show $\exists H \in \mathrm{SM}_{m+n}(U,X)$ such that

$$\forall u \in U, \qquad H(u,\ldots,u) = (P*Q)(u).$$

Choose $F \in \mathrm{SM}_m(U,V)$ such that

$$\forall u \in U, \qquad F(u,\ldots,u) = P(u).$$

Choose $G \in \mathrm{SM}_n(U,W)$ such that

$$\forall u \in U, \qquad G(u,\ldots,u) = Q(u).$$

Define $H_0 \in \mathcal{M}_{m+n}(U,\ldots,U,X)$ by

$$H_0(t_1,\ldots,t_m,u_1,\ldots,u_n) = [F(t_1,\ldots,t_m)] * [G(u_1,\ldots,u_n)].$$

Let $H := \mathrm{Sym}[H_0]$.

Homework 10: Due on Tuesday 4 April

38. Let X be a topological space, let W be a finite dimensional vector space, let $\phi : X \dashrightarrow W$, let $x \in X$ and let $\| \bullet \| \in \mathcal{N}(W)$. Show: $[\,\lim\limits_x \phi = 0_W\,] \;\Rightarrow\; [\,\lim\limits_x \|\phi\| = 0\,]$.

39. Let W be a finite dimensional vector space and let $\varepsilon \in \mathcal{O}_{\mathbb{R}W}(1)$. Show that $\lim\limits_{h\to 0} \dfrac{\varepsilon(h)}{h} = 0_W$, *i.e.*, show that $\lim\limits_0 \dfrac{\varepsilon}{\mathrm{id}_{\mathbb{R}}} = 0_W$.

40. Let V and W be finite dimensional vector spaces, let $f : V \dashrightarrow W$ and let $x \in \mathrm{dlin}[f]$. Show: $x \in \mathrm{dct}[f]$, *i.e.*, show: f is continuous at x.

Hint: Let $L := D_x f$ and let $R := f_x^T - L$. Then $L \in \mathcal{L}(V,W)$ and $R \in \mathcal{O}_{VW}(1)$. It follows that L and R are both continuous at 0_V. So, since $f_x^T = L + R$, we see that f_x^T is continuous at 0_V. Use this to show that f is continuous at x.

41. Let $|\bullet| \in \mathcal{N}(\mathbb{R})$ be the absolute value function. Show: $0 \notin \mathrm{dlin}[|\bullet|]$.

42. Let V be a finite dimensional vector space, let $f : V \dashrightarrow \mathbb{R}$ and let $x \in \mathrm{dlin}[f]$. Assume that $D_x f \neq \mathbf{0}_{V\mathbb{R}}$. Show that f does not have a local semi-max at x. That is, show, for any nbd U in V of x, that there exists $y \in U$ such that $f(y) > f(x)$.

Hint: Let U be given, and we seek y. Let $L := D_x f$ and let $R := f_x^T - L$. Then $L \in \mathcal{L}(V, \mathbb{R})$ and $R \in \mathcal{o}_{V\mathbb{R}}(1)$. Show that you can choose $v \in V$ such that $L(v) > 0$. Show that you can choose $h > 0$ small enough so that all that of the following works. Let $y := x + hv$. Then $y \in U$, and we wish to show: $f(y) > f(x)$. We have $|R(hv)| < h \cdot [L(v)]/100$. Then $f_x^T(hv) = h \cdot [L(v)] + [R(hv)] > 99 \cdot h \cdot [L(v)] / 100 > 0$. Then $[f(y)] - [f(x)] = f_x^T(hv) > 0$, so $f(y) > f(x)$, as desired.

43. Let V and W be finite dimensional vector spaces, $C \in \mathcal{C}(V, W)$. Let $X := \mathcal{L}(V, W)$. Show $DC = \mathbf{0}_{VX}$. That is, show: $\forall u \in V$, $D_u C = 0_X$.

Note: $0_X = \mathbf{0}_{VW}$.

Homework 9: Due on Tuesday 28 March

34. Let V and W be finite dimensional vector spaces. Prove that $\mathcal{P}_1(V, W) \subseteq \mathcal{O}_{VW}(1)$.

35. Let V and W be finite dimensional vector spaces. Prove that $[\mathcal{P}_1(V, W)] \cap [\mathcal{o}_{VW}(1)] = \{\mathbf{0}_{VW}\}$.

36. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume that $f' \in \mathcal{o}(3)$ and that $f(0) = 0$. Show that $f \in \mathcal{o}(4)$.

Hint: Using the Choice MVT, show that there exists $c \in \mathcal{O}(1)$ such that, for all $x \approx 0$, we have $f(x) = [f'(c(x))] \cdot x$.

37. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^3$. Show that $[D_2 f]_{11} = f'(2)$.

Homework 8: Due on Tuesday 21 March

31. Let V and W be finite dimensional vector spaces. Let $|\bullet| \in \mathcal{N}(V)$, $\varepsilon : V \dashrightarrow W$, $p > 0$. Assume $0_V \in \operatorname{dom}[\varepsilon]$ and $\varepsilon(0_V) = 0_W$. Show:

$$(\quad \varepsilon \in \mathcal{o}_{VW|\bullet|}(p) \quad) \qquad \Leftrightarrow \qquad \left(\quad \frac{\varepsilon}{|\bullet|^p} \in \mathcal{o}_{VW}^{\times} \quad \right).$$

32. Let V and W be finite dimensional vector spaces. Let $|\bullet| \in \mathcal{N}(V)$, $\alpha : V \dashrightarrow W$, $p > 0$. Assume $0_V \in \operatorname{dom}[\alpha]$ and $\alpha(0_V) = 0_W$. Show:

$$(\quad \alpha \in \mathcal{O}_{VW|\bullet|}(p) \quad) \qquad \Leftrightarrow \qquad \left(\quad \frac{\alpha}{|\bullet|^p} \in \mathcal{O}_{VW}^{\times} \quad \right).$$

33. Let V and W be finite dimensional vector spaces. Prove that $(\mathcal{O}_{V\mathbb{R}}^{\times}) \cdot (\mathcal{o}_{VW}^{\times}) \subseteq \mathcal{o}_{VW}^{\times}$.

Homework 7: Due on Tuesday 7 March

26. Let R, S and T be sets. Let $f : R \to S$ and let $g : S \to T$. Let $U \subseteq T$. Show: $f^*(g^*(U)) = (g \circ f)^*(U)$.

27. Let S and T be sets. Let $f : S \subset\!\!\to\!\!> T$. Let $U \subseteq S$. Show: $f_*(U) = (f^{-1})^*(U)$.

28. Let S and T be sets. Let $f : S \subset\!\!\to T$. Let $U \subseteq S$. Show: $f^*(f_*(U)) = U$.

29. Let S and T be sets. Let $f : S \to\!\!> T$. Let $U \subseteq T$. Show: $f_*(f^*(U)) = U$.

30. Let V be a vector space, $|\bullet| \in \mathcal{N}(V)$, $C > 0$. Let $x \in V$, $r > 0$. Show: $B_{|\bullet|}(x,r) = B_{C|\bullet|}(x,Cr)$.

Homework 6: Due on Tuesday 28 February

22. Let V be a vector space and let $* \in \mathrm{SBF}_{\geqslant 0}(V)$. Show both

- $\forall c \in \mathbb{R}, \forall x \in V, \quad |cx|_* = |c| \cdot |x|_*,$ and
- $|0_V|_* = 0.$

23. Let S be a set, let $n \in \mathbb{N}$, let V be a vector space and let $f : S^n \to V$. Let $g := \mathrm{Sym}[f] : S^n \to V$. Show both

(i) g is symmetric, *i.e.*, for all $x_1, \ldots, x_n \in S$, for all $\sigma \in \Sigma_n$, we have $g(x_1, \ldots, x_n) = g(x_{\sigma_1}, \ldots, x_{\sigma_n})$, and
(ii) the diagonal restrictions of f and g are the equal to one another, *i.e.*, for all $x \in S$, we have $g(x, x, \ldots, x) = f(x, x, \ldots, x)$.

24. Let V be a vector space. Show: $[\mathcal{P}_2(V)][\mathcal{P}_3(V)] \subseteq \mathcal{P}_5(V)$.

Hint: Given $f(v) = B(v,v)$ and $g(v) = T(v,v,v)$, we wish to show that $[f(v)][g(v)] = Q(v,v,v,v,v)$. (You need to set up all the quantifications.) We know that $[f(v)][g(v)] = [B(v,v)][T(v,v,v)]$. Let

$$Q_0(v,w,x,y,z) \quad = \quad [B(v,w)]\,[T(x,y,z)].$$

Let $Q := \mathrm{Sym}[Q_0]$.

25. Let V, W be vector spaces. Show: $[\mathcal{P}_2(W)] \circ [\mathcal{P}_3(V,W)] \subseteq \mathcal{P}_6(V)$.

Hint: Given $g(w) = B(w, w)$ and $f(v) = T(v, v, v)$, we wish to show that $g(f(v)) = S(v, v, v, v, v, v)$. (You need to set up all the quantifications.) We know that $g(f(v)) = B(T(v, v, v), T(v, v, v))$. Let

$$S_0(u, v, w, x, y, z) \quad = \quad B(\, T(u, v, w)\, ,\, T(x, y, z)\,).$$

Let $S := \mathrm{Sym}[S_0]$.

Homework 5: Due on Tuesday 21 February

18. For all $p \in (0, \infty]$, define $\overline{B}_p := \{x \in \mathbb{R}^2 \text{ s.t. } |x|_p \leqslant 1\}$. Then, for all $p \in (0, \infty)$, we have $\overline{B}_p = \{(s, t) \in \mathbb{R}^2 \text{ s.t. } |s|^p + |t|^p \leqslant 1\}$. Also, $\overline{B}_\infty = \{(s, t) \in \mathbb{R}^2 \text{s.t.} \max\{|s|, |t|\} \leqslant 1\}$. Graph $\overline{B}_{1/2}, \overline{B}_1, \overline{B}_2, \overline{B}_3, \overline{B}_4, \overline{B}_\infty$.

19. Show, for all $x \in \mathbb{R}^2$, that

$$|x|_1 \quad \geqslant \quad |x|_2 \quad \geqslant \quad |x|_\infty \quad \geqslant \quad |x|_1/100.$$

That is, show, for all $s, t \in \mathbb{R}$, that

$$|s| + |t| \;\; \geqslant \;\; \sqrt{s^2 + t^2} \;\; \geqslant \;\; \max\{|s|, |t|\} \;\; \geqslant \;\; (|s| + |t|)/100.$$

20. Find the largest $C > 0$ such that, $\forall x \in \mathbb{R}^2$, $|x|_\infty \geqslant C|x|_1$. That is, find the largest $C > 0$ such that, $\forall s, t \in \mathbb{R}$, $\max\{|s|, |t|\} \geqslant C(|s| + |t|)$.

21. Let $a, b, c \in \mathbb{R}$. Assume $a \geqslant 0$. Assume, for all $x \in \mathbb{R}$, that $ax^2 + 2bx + c \geqslant 0$. Show

(i) $(a = 0) \Rightarrow (b = 0)$, and
(ii) $ac - b^2 \geqslant 0$.

Hint for (ii): Replacing $x :\to -b/a$ in the assumption, we see that $a(-b/a)^2 + 2b(-b/a) + c \geqslant 0$.

Homework 4: Due on Tuesday 14 February

13. Let $m, n \in \mathbb{N}$, $L \in \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n)$, $v \in \mathbb{R}^m$. Show that $(L(v))^V = [L] \cdot v^V$.

(*Note:* We have $L(v) \in \mathbb{R}^n$, so $(L(v))^V \in \mathbb{R}^{n \times 1}$. Also, $[L] \in \mathbb{R}^{n \times m}$. Also, $v \in \mathbb{R}^m$, so $v^V \in \mathbb{R}^{m \times 1}$.)

14. Let $m, n \in \mathbb{N}$. Show that the two maps

$$\begin{array}{lcccccc} & L & \mapsto & [L] & : & \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n) & \to \mathbb{R}^{n \times m} \\ \text{and} & A & \mapsto & L_A & : & \mathbb{R}^{n \times m} & \to \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n) \end{array}$$

are inverses.

15. Let $m, n \in \mathbb{N}$, let $u \in \mathbb{R}^m$, let $v \in \mathbb{R}^n$ and let $A \in \mathbb{R}^{n \times m}$. Show that $B_A(u, v) = (v^H \cdot A \cdot u^V)_{11}$.

16. Let $m, n \in \mathbb{N}$. Show that the two maps

$$\begin{array}{rrcll} & B & \mapsto & [B] & : & \mathcal{B}(\mathbb{R}^m, \mathbb{R}^n) \to \mathbb{R}^{n \times m} \\ \text{and} & A & \mapsto & B_A & : & \mathbb{R}^{n \times m} \to \mathcal{B}(\mathbb{R}^m, \mathbb{R}^n) \end{array}$$

are inverses.

17. Let $n \in \mathbb{N}$. Show:

(i) $\forall B \in \mathrm{SBF}(\mathbb{R}^n), \quad [B] \in \mathbb{R}^{n \times n}_{\mathrm{sym}}$ and

(ii) $\forall A \in \mathbb{R}^{n \times n}_{\mathrm{sym}}, \quad B_A \in \mathrm{SBF}(\mathbb{R}^n)$.

Homework 3: Due on Tuesday 7 February

8. Let X be a topological space, let $f : \mathbb{R} \dashrightarrow X$ and let $a, b \in \mathbb{R}$. Show that $\lim\limits_{b} f \circ (a + \bullet) =^* \lim\limits_{a+b} f$.

9. Let X and Y be topological spaces. Let $f, g : X \dashrightarrow Y$. Assume that $f \subseteq g$; that is, assume, for all $x \in \mathrm{dom}\,[f]$, that both $x \in \mathrm{dom}\,[g]$ and $f(x) = g(x)$. Let $a \in \mathrm{LP}_X(\mathrm{dom}\,[f])$. Assume Y is Hausdorff. Show that $\lim\limits_{a} f =^* \lim\limits_{a} g$.

10. Let X and Y be topological spaces, $\phi : X \dashrightarrow Y$, $p \in X$ and $q \in Y$. Assume that $\phi \to q$ near p. Define $\psi : (\mathrm{dom}\,[\phi]) \cup \{p\} \to Y$ by

$$\psi(x) = \begin{cases} \phi(x), & \text{if } x \neq p; \\ q, & \text{if } x = p. \end{cases}$$

Show that $\psi : X \to Y$ is continuous at p.

11. Define $r : \mathbb{R} \backslash \{0\} \to \mathbb{R}$ by $r(x) = 1/x$. Show, for all $x \in \mathbb{R} \backslash \{0\}$, that

$$r'(x) \quad = \quad \frac{-1}{x^2} \, .$$

12. Let $f, g : \mathbb{R} \dashrightarrow \mathbb{R}$. Let $a \in \mathrm{LP}_{\mathbb{R}}(\mathrm{dom}\,[f/g])$. Show that

$$(f/g)'(a) \quad =^* \quad \frac{[g(a)][f'(a)] \; - \; [f(a)][g'(a)]}{[g(a)]^2}.$$

Hint: Define $r : \mathbb{R} \backslash \{0\} \to \mathbb{R}$ by $r(x) = 1/x$. Then $f/g = f \cdot (r \circ g)$.

Homework 2: Due on Tuesday 31 January

4. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $I \subseteq \mathrm{dd}[f]$. Assume that I is an interval. Let $S := f'_*(I)$. Assume that $S \geqslant 0$. Show that f is semi-increasing on I. That is, show, for all $a, b \in I$, that $[\,(\,a < b\,) \Rightarrow (\,f(a) \leqslant f(b)\,)\,]$.

5. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $I \subseteq \mathrm{dd}[f]$. Assume that I is an interval. Let $S := f'_*(I)$. Assume that $S < 0$. Show that f is strictly decreasing on I. That is, show, for all $a, b \in I$, that $[\,(\,a < b\,) \Rightarrow (\,f(a) > f(b)\,)\,]$.

6. Let $\delta > 0$. Let $U := (-\delta, \delta)$. Define $\beta : U \to \mathbb{R}$ by $\beta(h) = \max\{0, h\}$. Show that $\beta \to 0$ near 0 in $\mathbb{R}$.

7. Show that $\mathcal{O} \cdot \mathcal{o} \subseteq \mathcal{o}$. That is, show: $\quad \forall \alpha \in \mathcal{O}, \forall \varepsilon \in \mathcal{o}, \quad \alpha\varepsilon \in \mathcal{o}$.

Homework 1: Due on Tuesday 24 January

1. Let $g : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $L \in \mathbb{R}$. Assume g is defined near < 0. Assume $g \leqslant 0$ near < 0. Assume $g \to L$ near 0. Show $L \leqslant 0$.

2. Let $m, b \in \mathbb{R}$. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = mx + b$. Show $f' = C_{\mathbb{R}}^m$.

3. Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $a \in \mathrm{dd}[f]$. Assume f is defined near a in $\mathbb{R}$. Assume f has a local semi-max at a in $\mathbb{R}$. Show $f'(a) = 0$.
