MATH 4604 Spring 2018, Midterm #1
Handout date: Thursday 22 February 2018
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR @umn.edu EMAIL ADDRESS:

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Then f **is semiincreasing** means ...

B. (5 pts) Let W be a vector space. Let $f : \mathbb{R} \dashrightarrow W$. Let $p \in \mathrm{dom}[f]$. Then $SS_f^p : \mathbb{R} \dashrightarrow \mathbb{R}$ is defined by $(SS_f^p)(h) = \cdots$

C. (5 pts) Let X be a topological space, $S \subseteq X$. Then $\mathrm{LP}_X S = \cdots$.

D. (5 pts) Let X be a topological space. Let $f : X \dashrightarrow \mathbb{R}$ and let $p \in X$. Then f has a **local maximum** at p in X means: ...

E. (5 pts) Let X be a topological space. Then X is **sequentially compact** means: ...

F. (5 pts) Let S be a set. Let $f : S \to \mathbb{R}$ and let $p \in S$. Then f is **has an extremum at** p means ...

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ and assume that $f'(1) = 5$. Then there exists a neighborhood U of 1 in $\mathbb{R}$ such that f is strictly increasing on U.

b. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$. Assume f has a maximum at 0. Then f has a local maximum at 0.

c. (5 pts) Let X and Y be topological spaces, and let $f : X \to Y$. Let $w \in X$ and let $z \in Y$. Assume: $f \to z$ near w. Then $\lim_w f = z$.

d. (5 pts) Let $f : \mathbb{R} \hookrightarrow\!\!> \mathbb{R}$. Assume that $f : \mathbb{R} \to \mathbb{R}$ is continuous. Then $f^{-1} : \mathbb{R} \to \mathbb{R}$ is continuous.

e. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, and assume that f is strictly increasing. Then $\mathrm{im}[DQ_f] > 0$.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Give an example of two functions $f, g : \mathbb{R} \dashrightarrow \mathbb{R}$ such that:
[$f'(0) = 3$] and [$g'(0) = 4$] and [$(f + g)'(0) = ☹$].

2. (10 pts) Let $\| \bullet \| \in \mathcal{N}(\mathbb{R}^2)$ be defined by $\|(x,y)\| = \sqrt{x^2+y^2}$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = 7x - 8y$. Let $q := (3,2) \in \mathbb{R}^2$. Show: $\forall \varepsilon > 0$, $\exists \delta > 0$ s.t., $\forall p \in \mathbb{R}^2$,

$$[\ \| p - q \| < \delta\ ] \quad \Rightarrow \quad [\ |\,[f(p)] - [f(q)]\,| < \varepsilon\ ].$$

DON'T use theorems about continuity and limits.
Instead, follow templates: Give a method for finding δ, given ε,
then show that it works.

3. (10 pts) Define $f : \mathbb{R} \to \mathbb{R}^2$ by $f(x) = (2x + 4, 5x - 1)$. Let $p \in \mathbb{R}$. Prove that $f'(p) = (2, 5)$.

You may use without proof: For any topological spaces X and Y, for any $w \in \mathrm{LP}_X X$, for any $z \in Y$, $$\lim_w C_X^z = z.$$

You may also use without proof: For any topological spaces X and Y, for any $f, g : X \dashrightarrow Y$, for any $w \in X$,
$$[\ \exists U \in \mathcal{N}_X^\times(w) \text{ s.t. } (\ f = g \text{ on } U\ )\ ] \ \Rightarrow\ [\ \lim_w f = \lim_w g\ ].$$

4. (15 pts.) Let $f : \mathbb{R} \to \mathbb{R}$. Assume $f' = f$. Let $P : \mathbb{R} \to \mathbb{R}$ be defined by $P(x) = x^4$. Let $g := P \circ f$. Let $a \in \mathbb{R}$. Show: $g'(a) = 4 \cdot [g(a)]$.

If you invoke the Chain Rule, be sure to verify its limit point hypothesis and don't forget that its conclusion is a *contingent* equality. You may use, without proof, that: $\quad \forall x \in \mathbb{R}, \quad P'(x) = 4x^3$.