MATH 4604 Spring 2018, Midterm #2
Handout date: Thursday 22 March 2018
Instructor: Scot Adams

PRINT YOUR NAME:

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Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $S \subseteq \mathbb{R}$. Then f is **c/d** on S means: ...

B. (5 pts) Let X and Y be topological spaces, and let $f : X \dashrightarrow Y$. Then f is **closed** means: ...

C. (5 pts) Let X be a topological space, let $p \in X$, and let $f : X \dashrightarrow \mathbb{R}$. Then f has a **local maximum** at p in X means: ...

D. (5 pts) Let V be a vector space and let $\| \bullet \| : V \to [0, \infty)$. Then $\| \bullet \|$ is a **norm on** V means: ...

E. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$. Then f is **semimonotone** means: ...

F. (5 pts) Let X and Y be topological spaces, and let $f : X \dashrightarrow Y$. Then $\mathrm{dct}[f] := \cdots$.

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ be 1-1 and continuous. Then f^{-1} is continuous.

b. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be 1-1 and continuous. Then f is strictly monotone.

c. (5 pts) Let $f : [1,2] \to [3,4]$ be continuous. Then f is closed.

d. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Assume that $f'(0) = 1$. Then there exists $\delta > 0$ such that f is strictly increasing on $(-\delta, \delta)$.

e. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$ both be differentiable. Assume that $f' = g'$. Assume that $f(0) = g(0)$. Then $f = g$.

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I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Find a function $f : \mathbb{R} \to \mathbb{R}$ such that $f(0) = 2$, $f'(0) = 3$, $f''(0) = 4$ and $f'''(0) = 6$.

2. (10 pts) Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x^5 + x^3 + x$. Let $g := f^{-1}$. Note that $g(3) = 1$. Compute $g'(3)$.

3. (10 pts) Let $S := (0, \infty)$. Recall: $\exp : \mathbb{R} \hookrightarrow\!\!\!> S$ is defined by

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots .$$

Let $\ln := \exp^{-1} : S \to \mathbb{R}$. Let $q \in S$. Show: $\ln'(q) = 1/q$.

Hint: Recall that $\exp' = \exp$.

4. (15 pts.) Let $f : \mathbb{R} \to \mathbb{R}$. Assume: $\forall t \in \mathbb{R}, \quad f''(t) \;=\; -\dfrac{f(t)}{1 + [f(t)]^2}$.

Define $P, K : \mathbb{R} \to \mathbb{R}$ by $P(t) = \ln(1 + [f(t)]^2)$ and $K(t) = [f'(t)]^2$. Show that $P + K$ is constant.