MATH 4604 Spring 2019, Final exam
Handout date: Monday 13 May 2019
Instructor: Scot Adams

PRINT YOUR NAME:

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Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let X be a metric space. Then X is **proper** means ...

B. (5 pts) Let X be a metric space. Then X is **complete** means ...

C. (5 pts) Let $V, W \in$ TNSR and let $L \in \mathcal{L}_V^W$. Then $[L]_V^W \in W \otimes V$ is defined by: $\forall i \in \mathcal{I}_V$, $\forall j \in \mathcal{I}_W$, $([L]_V^W)_{j\|i} = \cdots$

D. (5 pts) Let $V, W \in \mathrm{TNSR}$, $f : V \dashrightarrow W$, $p \in V$.
Then $\mathrm{LINS}_p^{VW} f := \cdots$.

E. (5 pts) Let $V, W \in \mathrm{TNSR}$, $f : V \dashrightarrow W$, $p \in V$.
Then $D_p^{VW} f := \cdots$.

F. (5 pts) Let μ be a function.
Then μ is a **partition measure** means ...

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable. Assume: f is one-to-one. Then: $\forall x \in \mathbb{R},\ f'(x) \neq 0$.

b. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}^2$. Assume: $\forall x \in \mathbb{R},\ f'(x) \neq (0, 0)$. Then f is one-to-one.

c. (5 pts) Let $f : [1, 2] \cup (3; 4] \hookrightarrow \mathbb{R}$. Assume that f is continuous. Then f^{-1} is continuous.

d. (5 pts) $\forall f, g \in \widehat{\mathcal{O}}_0, \quad g \circ f \in \widehat{\mathcal{O}}_0$.

e. (5 pts) Let $\mathcal{P}$ be a partition and let $B \subseteq \bigcup \mathcal{P}$. Then $B^-_{\mathcal{P}} \subseteq B \subseteq B^+_{\mathcal{P}}$.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Find a function $f : \mathbb{R}^2 \to \mathbb{R}$ s.t.

$$(\partial_1 f)(0_2) = 0 \quad \text{and} \quad (\partial_2 f)(0_2) = 0 \quad \text{and} \quad f'(0_2) = ☹.$$

2. (10 pts) Let $V, Z \in$ TNSR, $K \in \mathcal{K}_V^Z$.
Show: $\exists C \geqslant 0$ s.t., $\forall p \in V$, $|K(p)|_Z \leqslant C \cdot |p|_V^3$.

3. (10 pts) Let $f : \mathbb{R}^2 \to \mathbb{R}$, $u, v \in \mathbb{R}^2$.
Assume: $\forall p \in \mathbb{R}^2$, $(\partial_u f)(p) = 0$. Show: $f(v) = f(v + u)$.

4. (15 pts.) Let $f : \mathbb{R}^2 \to \mathbb{R}$ and let $v, w \in \mathbb{R}^2$.
Assume: $\forall p \in \mathbb{R}^2,\ f'(p) = 0_2$. Show: $f(v) = f(w)$.