MATH 4604 Spring 2019, Midterm #1
Handout date: Thursday 28 February 2019
Instructor: Scot Adams

PRINT YOUR NAME:

PRINT YOUR @umn.edu EMAIL ADDRESS:

One hour exam.

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $S \subseteq \mathrm{dom}[f]$.
Then f **is strictly decreasing** on S means ...

B. (5 pts) Let Z be a metric space and let $f : Z \dashrightarrow \mathbb{R}$.
Then f has a **strict local minimum** at p in Z means: ...

C. (5 pts) CVZ := $\cdots$.

D. (5 pts) Let $k \in \mathbb{N}_0$. Then $\widehat{\mathcal{O}}_k := \cdots$.

E. (5 pts) Let X be a metric space. Then X is **compact** means: ...

F. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $p \in \mathbb{R}$. Then $f'(p) := \cdots$.

II. True or false (no partial credit):

a. (5 pts) Let $f:\mathbb{R}\to\mathbb{R}$. Assume $\mathrm{dom}[f'] = \mathbb{R}$.
Assume that f is strictly increasing on $\mathbb{R}$. Then $f' > 0$ on $\mathbb{R}$.

b. (5 pts) Let $f:\mathbb{R}\to\mathbb{R}$. Assume $\mathrm{dom}[f'] = \mathbb{R}$.
Assume $f(0) = \max(\mathrm{im}[f])$. Then $f'(0) = 0$.

c. (5 pts) Let $f:\mathbb{R}\to\mathbb{R}$.
Assum both $f(0) = 0$ and $f' = C^0_{\mathbb{R}}$. Then $f = C^0_{\mathbb{R}}$.

d. (5 pts) Let $f:\mathbb{R} \dashrightarrow \mathbb{R}$ and let $p\in\mathbb{R}$. Then $\#(\mathrm{LINS}_p f) \leqslant 1$.

e. (5 pts) We have $(\widehat{\mathcal{O}}_3)\cdot(\mathcal{O}_3) \subseteq \mathcal{O}_9$.

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I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Give an example of two functions $f, g : \mathbb{R} \to \mathbb{R}$ such that:

$$(\ \ 0 \notin \mathrm{dom}[f']\ \ ) \ \ \&\ \ (\ \ 0 \notin \mathrm{dom}[g']\ \ ) \ \ \&\ \ (\ \ (f+g)'(0) = 1\ \ ).$$

2. (10 pts) Let $f, \gamma, h : \mathbb{R} \dashrightarrow \mathbb{R}$, let $p \in \mathbb{R}$ and let $q := f_p$.

Show: $((\gamma \circ f) \cdot h)'_p \;\; =^* \;\; (\gamma'_q) \cdot (f'_p) \cdot (h_p) + (\gamma_q) \cdot (h'_p)$.

3. (10 pts) Let $f, g : \mathbb{R} \dashrightarrow \mathbb{R}$, let $p \in \mathrm{dom}[f'']$ and let $q := f_p$.
Assume: $q \in \mathrm{dom}[g'']$. Prove: $(g \circ f)''_p = (g''_q) \cdot (f'_p)^2 + (g'_q) \cdot (f''_p)$.

Hint: Let $\phi := (g \circ f)'$ and $\psi := (g' \circ f) \cdot f'$. Show: $\phi = \psi$ near p.
You may then conclude, without proof, that $\phi'_p = \psi'_p$.
To compute ψ'_p, use Problem 2 (replacing γ by g' and h by f').

4. (15 pts.) Let $g : \mathbb{R} \dashrightarrow \mathbb{R}$. Assume g' is c/d on $[0; 3]$. Assume $g_0 = g'_0 = 0 = g_3$. Show: $\exists c \in (0; 3)$ s.t. $g''_c = 0$.