MATH 4604 Spring 2019, Midterm #2
Handout date: Thursday 28 March 2019
Instructor: Scot Adams

PRINT YOUR NAME:

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Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) Let $V, W \in \mathrm{TNSR}^+$. Let $f : V \dashrightarrow W$, $S \subseteq V$. Then f is **c/d** on S means: ...

B. (5 pts) Let $V, W, X \in \mathrm{TNSR}^+$. Then $\mathcal{B}^X_{VW} \ := \ \cdots$.

C. (5 pts) Let Z be a metric space, let $p \in Z$, and let $f : Z \dashrightarrow \mathbb{R}$. Then f has a **strict local maximum** at p in Z means: ...

D. (5 pts) Let $V, W, X \in \mathrm{TNSR}^+$, $A \in W \otimes V$, $B \in X \otimes W$.
Then $B \; *_{VWX} \; A \quad := \quad \cdots$

E. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $S \subseteq \mathbb{R}$.
Then f is **semi-decreasing** on S means: . . .

F. (5 pts) Let $\sigma \in \mathbb{N}$. Then $\mathrm{TNSR}_\sigma \; := \; \cdots$.

II. True or false (no partial credit):

a. (5 pts) Let $f : [0,1] \hookrightarrow\!\!> [3,7]$. Assume f is continuous. Then f^{-1} is continuous.

b. (5 pts) Let $f, g : \mathbb{R} \to \mathbb{R}$. Assume that $0 \in \mathrm{dom}[(g \circ f)']$. Then $0 \in \mathrm{dom}[f']$ and $f(0) \in \mathrm{dom}[g']$.

c. (5 pts) Let $V, W \in \mathrm{TNSR}^+$, $f : V \dashrightarrow W$, $p, q \in V$. Then $(f(p + \bullet))'(q) = f'(p + q)$.

d. (5 pts) We have $\mathcal{O}_2 \circ \widehat{\mathcal{O}}_4 \subseteq \mathcal{O}_8$.

e. (5 pts) We have $\mathcal{O}_2 \cdot \widehat{\mathcal{O}}_4 \subseteq \mathcal{O}_8$.

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I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Find $f \in \mathcal{O}_0$ and $g \in \widehat{\mathcal{O}}_0$ s.t. $g \circ f \notin \mathcal{O}_0$.

Hint: $\mathcal{O}_0 =$ CVZ and $\widehat{\mathcal{O}}_0 =$ BNZ.

2. (10 pts) Let $V := \mathbb{R}^3$, $W := \mathbb{R}$.
Define $K \in \mathcal{K}_V^W$ by $K(x, y, z) = x^3 + 6y^2z$.
Let $p := (1, 2, 3)$. Compute K'_p.

3. (10 pts) Let $V, W, X \in \text{TNSR}$, $B \in X \otimes W$, $A \in W \otimes V$.
Show: $|B \; *_{VWX} \; A|_{X \otimes V} \leqslant |B|_{X \otimes W} \cdot |A|_{W \otimes V}$.

Hint: $\forall k \in \mathcal{I}_X$, $\forall i \in \mathcal{I}_V$, $(B \; *_{VWX} \; A)_{k\|i} = \sum_{j \in \mathcal{I}_W} B_{k\|j} \cdot A_{j\|i}$.

Use Cauchy-Schwarz.

4. (15 pts.) Let $\quad f : \mathbb{R} \to \mathbb{R}^2 \quad$ and $\quad g : \mathbb{R}^2 \to \mathbb{R}$.

Assume: $f_0 = (1,2)$ and $g'_{(1,2)} = (0,0)$ and $g''_{(1,2)} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Assume: $0 \in \mathrm{dom}[f'']$. $\qquad$ Show that $(g \circ f)''_0 \geqslant 0$.

Hint: $$(g \circ f)''_0 \quad =^* \quad (g''_{f_0}) \bullet_{\mathbb{R}^{2\times 2}} ((f'_0)^{\otimes 2}) + (g'_{f_0}) \bullet_{\mathbb{R}^2} (f''_0).$$