MATH 4604 Spring 2020, Midterm #1
Handout date: Thursday 27 February 2020
Instructor: Scot Adams

PRINT YOUR NAME:

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One hour exam.

Closed book, closed notes, no calculators/PDAs; no reference materials of any kind.

Turn off all mobile electronic devices.

I. Definitions

A. (5 pts) BNZ $:= \cdots$.

B. (5 pts) Let $k \in \mathbb{N}_0$. Then $\widehat{\mathcal{O}}_k := \cdots$

C. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$, $x \in \mathbb{R}$. Then $\mathrm{LINS}_x f := \cdots$.

D. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $x \in \mathbb{R}$. Then $f'_x := \cdots$.

E. (5 pts) Let X be a metric space. Let $f : X \dashrightarrow \mathbb{R}$, $q \in X$.
By f has a **local strict-maximum** at q in X, we mean ...

F. (5 pts) Let $V \in \text{ES}$ and $x, y \in V$. Then $x \bullet y \; := \; \cdots$

II. True or false (no partial credit):

a. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$. Assume $\mathrm{dom}[f'] = \mathbb{R}$.
Assume that f is strictly-increasing on $\mathbb{R}$. Then $f' \geqslant 0$ on $\mathbb{R}$.

b. (5 pts) $\forall a, b \in \mathbb{R}$, $|a| + |b| \leqslant \sqrt{a^2 + b^2}$.

c. (5 pts) Let $f : \mathbb{R} \to \mathbb{R}$.
Assume both $f(0) = 0$ and $f' \in \mathcal{O}_4$. Then $f \in \mathcal{O}_5$.

d. (5 pts) Let $V, W \in \mathrm{ES}$ and let $L \in \mathcal{L}_W^V$.
Then L is Lipschitz from V to W.

e. (5 pts) Let $f : \mathbb{R} \dashrightarrow \mathbb{R}$ and let $x \in \mathbb{D}_f$.
Then: $\forall h \in \mathbb{R}$, $(f_x^{\mathbb{T}})'_h = f'_{x+h} - f'_x$.

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PLEASE DO NOT WRITE BELOW THE LINE

I. A,B,C

I. D,E,F

II. a,b,c,d,e

III. 1

III. 2

III. 3

III. 4

III. Hand-graded problems. Show work.

1. (10 pts) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by: $\forall x, y \in \mathbb{R}, \quad f(x, y) = 3x + 4y$.
   Show that f is Lipschitz-5.

2. (10 pts) Let $f, g : \mathbb{R} \dashrightarrow \mathbb{R}$.

Assume: $f_0 = f'_0 = g_0 = g'_0 = 0$.

Show: $f \cdot g \in \mathcal{O}_2$.

3. (10 pts) Let $a \in [0;\infty)^{\mathbb{N}}$. Assume: $\forall j \in \mathbb{N},\ a_{j+1} \leqslant a_j/2$.
Using Mathematical Induction, show: $\forall j \in \mathbb{N},\ \ a_j \leqslant a_1/2^{j-1}$.

4. (15 pts.) Let $a \in [0;\infty)^{\mathbb{N}}$. Assume: $\forall j \in \mathbb{N},\ \ a_{j+1} \leqslant a_j/2$.
Define $b \in [0;\infty)^{\mathbb{N}}$ by: $\forall j \in \mathbb{N},\ \ b_j = a_1 + a_2 + \cdots + a_j$.
Show that b is convergent in $\mathbb{R}$.

Hint: Use Problem 3, and remember that $\mathbb{R}$ is complete.