Central Limit Theorem and Finance
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Applied Coin-Flipping

\[ N = 10^{10^{100}} \]

\[ N \] coin flips

\[ \frac{H - T}{\sqrt{N}} \]

heads

tails

Male height (inches): 69 + 5 \[ \frac{H - T}{\sqrt{N}} \]

Probability that: 69 − 5 \leq ht \leq 69 + 5?

\[ 69 - 5 \leq 69 + 5 \]

\[ \frac{H - T}{\sqrt{N}} \]

\[ \leq 69 + 5 \]

\[ -5 \leq 5 \frac{H - T}{\sqrt{N}} \leq 5 \]

\[ -1 \leq \frac{H - T}{\sqrt{N}} \leq 1 \]

DIVIDE BY 5
Applied Coin-Flipping

\[ N = 10^{10^{100}} \]

\[ N \text{ coin flips} \quad \frac{H}{T} \text{ heads tails} \]

Male height (inches): \[ 69 + 5 \frac{H - T}{\sqrt{N}} \]

Probability that: \[ 69 - 5 \leq ht \leq 69 + 5? \]

Probability that: \[ -1 \leq \frac{H - T}{\sqrt{N}} \leq 1? \]

\[ -1 \leq \frac{H - T}{\sqrt{N}} \leq 1 \]
Applied Coin-Flipping

\[ N = 10^{10^{100}} \]

\[ N \text{ coin flips} \]

\[ \frac{H}{T} \text{ heads to tails} \]

**Male height (inches):** 69 + 5 \( \frac{H - T}{\sqrt{N}} \) square root

Probability that: 69 – 5 \( \leq \) ht \( \leq \) 69 + 5?

Probability that: \(-1 \leq \frac{H - T}{\sqrt{N}} \leq 1\)?

**Answer:** \( \approx 68\% \)

**Grav accel (ft/sec^2):** 32 + \( 10^6 \frac{H - T}{N} \) NO square root

Probability that: 32 – \( \frac{10^6}{\sqrt{N}} \) \( \leq \) acc \( \leq \) 32 + \( \frac{10^6}{\sqrt{N}} \)?

EXTREMELY small

Probability that: \(-1 \leq \frac{H - T}{\sqrt{N}} \leq 1\)?

**Answer:** \( \approx 68\% \)
Applied Coin-Flipping

\[ N = \text{number of seconds in 30 days} \]

Current stock price: 1 USD

\[ x_+ := \begin{cases} 
  x, & \text{if } x \geq 0 \\
  0, & \text{if } x \leq 0 
\end{cases} \quad S := \text{stock price} \]

30 days from now

Contract pays: \((S - 1)_+\) USD,

30 days from now

Expected payout?

Each second, price changes
either by a factor of 1.000035616
or by a factor of 0.999964386.

50% chance of uptick,
50% chance of downtick.
Applied Coin-Flipping

Coin-flipping game: Flip a fair coin \( N \) times. If \( H \) heads and \( T \) tails, pay \( (u^H d^T - 1)_+ \), 30 days from now.

Contract pays: \((S - 1)_+ \) USD, 30 days from now

Expected payout?

Each second, price changes either by a factor of 1.000035616 or by a factor of 0.999964386.

50% chance of uptick, 50% chance of downtick.
Applied Coin-Flipping

Coin-flipping game: Flip a fair coin $N$ times. If $H$ heads and $T$ tails, pay $(u^H d^T - 1)_+$, 30 days from now.

Expected payout?

Computing probabilities is relatively easy, computing expected values is generally harder.
Compute the probability that

\[-1 < \frac{H - T}{\sqrt{N}} < 1.\]

\[
X := \frac{H - T}{\sqrt{N}}
\]

Compute the probability that

\[-1 < X < 1.\]

\[
H_1 := \text{number of heads after first flip}
\]
\[
H_2 := \text{number of heads after second flip}
\]
\[
\vdots
\]
\[
H_N := \text{number of heads after } N\text{th flip} = H
\]
Compute the probability that
\[-1 < \frac{H - T}{\sqrt{N}} < 1.\]

\[
X := \frac{(H - T)}{\sqrt{N}}
\]

Compute the probability that
\[-1 < X < 1. \quad X \text{ is hard} \ldots
\]

For all integers \(j \in [1, N]\),
\[
H_j := \text{number of heads after } j\text{th flip}
\]
\[
T_j := \text{number of tails after } j\text{th flip}
\]
\[
D_j := H_j - T_j
\]

Easier: \(D_1, D_1/7, D_2, D_N\)

\[
H = H_N, \quad T = T_N, \quad X = \frac{(H_N - T_N)}{\sqrt{N}} = \frac{D_N}{\sqrt{N}}
\]
\[ D_1 = H_1 - T_1 : \] random variable

a variable whose value is determined by random events

(probability) measure of \( D_1 \)

(probability) distribution of \( D_1 \)

distribution of \( T_1 - H_1 \)
is exactly the same

keep the distribution
forget its origin
0.5 \times \begin{bmatrix} e^{it} \\ e^{-it} \end{bmatrix} = \begin{bmatrix} \cos t + i \sin t \\ \cos t - i \sin t \end{bmatrix} +

\text{Inverse Fourier transform}

\text{Cannot recover the random variable!}

\text{Fourier transform of the distribution of } D_1 \text{ is } \cos t

\text{Repl. } t \text{ by } t/7

\text{What about } D_1/7?

\text{Generating function:}

\text{Fourier transform:}

\text{keep the distribution} \not\text{forget its origin}

z^{-1}

divide by 7

T_1 - H_1 \text{ has the same distribution.}

\text{Replace } z \text{ by } e^{\omega t}

\left(0.5\right) z + \left(0.5\right) z^{-1}

\left(0.5\right) e^{-it} + \left(0.5\right) e^{it}

\| \cos t
What about $D_{1/7}$?
Replace $t$ by $t/7$.

$i = \sqrt{-1}$

Replace $z$ by $e^{-it}$

Generating function:
$$(0.5)z^{1/7} + (0.5)z^{-1/7}$$

Fourier transform:
$$(0.5)e^{-it/7} + (0.5)e^{it/7} \equiv \cos(t/7)$$

\[
e^{it/7} = \cos(t/7) + i \sin(t/7) \\
e^{-it/7} = \cos(t/7) - i \sin(t/7)
\]
\[ D_2 = H_2 - T_2 : \]

\[
\begin{array}{c|c}
& 0 & 2 \\
\hline
\downarrow \downarrow & \downarrow \downarrow & \downarrow \\
2 & & 0.25 \\
0 & & 0.25 + 0.25 = 0.5 \\
-2 & & 0.25 \\
\end{array}
\]

*forget its origin keep the distribution*
\[ D_2 = H_2 - T_2 : \]
\[
\begin{array}{c|cc}
 & 2 & 0 & -2 \\
0.25 & \rightarrow & \rightarrow & \rightarrow \\
0.5 & 0 & z^0 = 1 & \rightarrow \\
0.25 & -2 & z^{-2} & \rightarrow \\
\end{array}
\]

"forget its origin keep the distribution"

Generating function:
\[
(0.25)z^2 + 0.5 + (0.25)z^{-2} = ((0.5)z + (0.5)z^{-1})^2
\]

the generating function of the distribution of \( D_1 \)

Replace \( z \) by \( e^{-it} \)

Fourier transform:
\[
(cos t)^2 = \cos^2 t
\]
Fourier transform:

$$\cos t \rightarrow \cos N \frac{t}{N}$$

Replace $z$ by $e^{-it}$:

$$z = \sqrt{-1}$$

Replace $i$ by $\sqrt{-1}$:

$$i = \sqrt{-1}$$

Goal:

$$X = \sqrt{\frac{D_N}{N}}$$

What about $D_N/\sqrt{N}$?

Generating function:

$$\text{NO WAY!!}$$

$$= \left(0.5z + (0.5z-1)\right)_N$$

divide by $\sqrt{N}$

$$D_N = H_N \frac{1}{\sqrt{N}}$$

the generating function of the distribution of $D_1$
\[ X = D_N / \sqrt{N} \]

Goal: \( X = D_N / \sqrt{N} \)?
Replace \( t \) by \( t / \sqrt{N} \).

Fourier transform:
\[ \cos^N(t / \sqrt{N}) \]
$X = \frac{D_N}{\sqrt{N}} :$ NO WAY!

Fourier transform: $\cos^N(t/\sqrt{N})$

Generating functions Fourier transforms
\[ X = D_N / \sqrt{N} : \]

The problem:

\textbf{Compute} the probability that \(-1 < X < 1\).

Exercise: \( \lim_{n \to \infty} \cos^n \left( \frac{3}{\sqrt{n}} \right) = e^{-3^2/2} \)

Fourier transform:

\[ \cos^N \left( \frac{t}{\sqrt{N}} \right) \]

\[ \approx \lim_{n \to \infty} \cos^n \left( \frac{t}{\sqrt{n}} \right) = e^{-t^2/2} \]

\textbf{Verify for} \( t = 3 \).
\[ X = D_N / \sqrt{N} \]

Fourier transform:

\[ \cos^N \left( \frac{t}{\sqrt{N}} \right) \]

\[ \approx \lim_{n \to \infty} \cos^n \left( \frac{t}{\sqrt{n}} \right) = e^{-t^2/2} \]
\[ X = \frac{D_N}{\sqrt{N}} : \]

Fourier transform:
\[ \cos^N(t/\sqrt{N}) \]

\[ \approx \lim_{n \to \infty} \cos^n(t/\sqrt{n}) \approx e^{-t^2/2} \]

**Key idea of Central Limit Theorem:**

Let \( Z \) have distr. with Fourier transf. \( e^{-t^2/2} \).

Then \( Z \) is “close” to \( X \).

**The problem:**

Compute the probability that
\[ -1 < X < 1. \]

Approximately equal to the probability that
\[ -1 < Z < 1. \]
\[ Z: \int_{\text{infinitesimal}}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx \quad \text{for all } x \in \mathbb{R} \]

**NOTES**

**Mistake:**

\[
\int_{-\infty}^{\infty} e^{-x^2/2} \, dx = \sqrt{2\pi}
\]

\[ D_2 \in \{2, 0, -2\} \]

distribution supported on three points

\[ D_N \in \{-N, -N + 2, \ldots, N - 2, N\} \]

distribution supported on \( N + 1 \) points

By contrast, the distribution of \( Z \) does **not** have finite support.
Problem: Compute the probability that \( Z = 7 \)

Solution: \[
\int_{7}^{7} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = 0
\]
Problem: Compute the probability that $2 < Z < 3$

Solution: \[
\int_{2}^{3} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = [\Phi(x)]_{x=2}^{x=3} = \Phi(3) - \Phi(2) = 0.0214 = 2.14\%
\]
$Z$: \[ \frac{1}{\sqrt{2\pi}} \int e^{-x^2/2} \, dx \]

Do this for all $x \in \mathbb{R}$

**Generating function:**

\[ \int_{-\infty}^{\infty} z^x \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = \text{Exercise} \]

**Fourier transform:**

\[ \int_{-\infty}^{\infty} e^{-itx} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = e^{-t^2/2} \]

**Key idea of Central Limit Theorem:**

Let $Z$ have distr. with Fourier transf. $e^{-t^2/2}$.

Then $Z$ is “close” to $X$. 
\[
Z: \quad \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx
\]

Do this for all \(x \in \mathbb{R}\)

**Exercise:**
\[
\int_{-\infty}^{\infty} e^{3x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = e^{3^2/2}
\]

**Fourier transform:**
\[
\int_{-\infty}^{\infty} e^{-itx} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = e^{-t^2/2}
\]

Verify for \(t = 3i\).

**Key idea of Central Limit Theorem:**
Let \(Z\) have distr. with Fourier transf. \(e^{-t^2/2}\).
Then \(Z\) is “close” to \(X\).
The problem:
Compute the probability that $-1 < X < 1$.

Approximately equal to the probability that $-1 < Z < 1$.

Approximate solution:

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx = [\Phi(x)]_{x \equiv 1}^{x \equiv -1} = 68.27\%$$
Goal: Compute the expected value of $f(u^H d^T)$.

Coin-flipping game: Flip a fair coin $N$ times. If $H$ heads and $T$ tails, pay $(u^H d^T - 1)_+$, 30 days from now.

$$f(x) = (x - 1)_+$$
Goal: 
Compute the expected value of $f(u^H d^T)$.

New easier problem: 
Compute the expected value of $f(D_2)$.

\[ D_2 = H_2 - T_2 : \]
\[
\begin{array}{c|c}
2 & f(2) \\
0 & f(0) \\
-2 & f(-2) \\
\end{array}
\]

\[
[0.25][f(2)] + [0.5][f(0)] + [0.25][f(-2)] = 1,250
\]
Define: \( g(x) = 5e^x + x^2 \)

Goal:

Compute the expected value of \( f(u^H d^T) \).

New easier problem:

Compute the expected value of \( g(D_2) \).

\[
D_2 = H_2 - T_2 : \\
\begin{array}{c|cc}
0.25 & 2 & g(2) \\
0.5 & 0 & g(0) \\
0.25 & -2 & g(-2) \\
\end{array}
\]

\[
[0.25][g(2)] + [0.5][g(0)] + [0.25][g(-2)] = \text{Exercise}
\]
Recall: \[ f(x) = (x - 1)_+ \]

Goal:
Compute the expected value of \( f(u^H d^T) \).

New easier problem:
Compute the expected value of \( f(Z) \).

\[ Z: \quad \frac{1}{\sqrt{2\pi}} \to e^{-x^2/2} \ dx \]

Do this for all \( x \in \mathbb{R} \)

\[
\int_{-\infty}^{\infty} [f(x)] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \ dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)] e^{-x^2/2} \ dx = \text{exercise}_{30}
\]
Recall: \( f(x) = (x - 1)_+ \)

Goal:

Compute the expected value of \( f(u^H d^T) \).

New easier problem:

Compute the expected value of \( f(X) \).

\[
\mathbb{E}
\begin{bmatrix}
\frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \\
X \\
Z
\end{bmatrix}
\begin{bmatrix}
x \\
f(x)
\end{bmatrix}
\text{Do this for all } x \in \mathbb{R}
\]

\[
\int_{-\infty}^{\infty} [f(x)] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx
\]

Approx. Sol’n: \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [f(x)] e^{-x^2/2} dx
\]

\[
f(x) = (5000x - 5000)_+ = \text{exercise}_3
\]
Recall: \[ f(x) = (x - 1)^+ \]
Goal:
Compute the expected value of \[ f(u^H d^T) \].
New easier problem:
Compute the expected value of \[ g(X) \].

Approx. Sol’n: \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} dx \]
Recall: \( f(x) = (x - 1)_+ \)

Goal:

Compute the expected value of \( f(u^H d^T) \).

New easier problem:

Compute the expected value of \( g(X) \).

\[
X = \frac{(H - T)/\sqrt{N}}{\times \sqrt{N}}
\]

\[
N = 2,592,000
\]

\[
H + T = N
\]

\[
H - T = X \sqrt{N}
\]

\[
2H = N + X \sqrt{N}
\]

\[
2T = N - X \sqrt{N}
\]

Approx. Sol’n: \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(x) = (x - 1)_+ \]

Goal: Compute the expected value of \( f(u^H d^T) \).

New easier problem: Compute the expected value of \( g(X) \).

\[
\begin{align*}
H &= N/2 + X\sqrt{N}/2 \\
T &= N/2 - X\sqrt{N}/2 \\
u^H &= u^{N/2} X\sqrt{N}/2 \\
d^T &= d^{N/2} d^{-X\sqrt{N}/2} \\
u^H d^T &= N_u \equiv 30 \times 24 \times 60 \times 60 = 2,592,000 \\
2H &= N + X\sqrt{N} \\
2T &= N - X\sqrt{N}
\end{align*}
\]

Approx. Sol’n: \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]

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Recall: \( f(x) = (x - 1)_+ \)

Goal:
Compute the expected value of \( f(u^H d^T) \).

New easier problem:
Compute the expected value of \( g(X) \).

\[
\begin{align*}
H &= N/2 + X\sqrt{\bar{N}}/2 \\
T &= N/2 - X\sqrt{\bar{N}}/2 \\

u^H &= u^{N/2} u X \sqrt{\bar{N}}/2 \\
d^T &= d^{N/2} d^{-X\sqrt{\bar{N}}/2} \\

u^H d^T &= u^{N/2} d^{N/2} u X \sqrt{\bar{N}}/2 d^{-X\sqrt{\bar{N}}/2} \\
&= (ud)^{N/2} (u/d)^X \sqrt{\bar{N}}/2 \\
&= C \ e^{kX} \\

C &:= (ud)^{N/2} \\
k &:= \ln((u/d)^{\sqrt{\bar{N}}/2})
\end{align*}
\]

Approx. Sol’n:
\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] \ e^{-x^2/2} \ dx = e^k = (u/d)^{\sqrt{\bar{N}}/2}
\]
Recall: \( f(x) = (x - 1)_+ \)

Goal:

Compute the expected value of \( f(u^H d^T) \).

New easier problem:

Compute the expected value of \( g(X) \).

\[
f(u^H d^T) = f(Ce^{kX}) = g(X)
\]

\[
u^H d^T = u^{N/2} d^{N/2} u^{X \sqrt{N}/2} d^{-X \sqrt{N}/2}
\]

\[
= (ud)^{N/2} (u/d)^X \sqrt{N}/2
\]

\[
= C e^{kX}
\]

\[
k := \ln((u/d)^{\sqrt{N}/2})
\]

Approx. Sol’n:

\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \[ f(x) = (x - 1)_+ \]

Goal:
Compute the expected value of \( f(u^H d^T) \).

Restatement of goal:
Compute the expected value of \( g(X) \).

\[
f(u^H d^T) = f(C e^{kX}) = g(X) \quad \text{with} \quad g(x) := f(C e^{kx})
\]

\[
u^H d^T = u^{N/2} d^{N/2} u^{X\sqrt{N}/2} d^{-X\sqrt{N}/2} = (ud)^{N/2} (u/d)^{X\sqrt{N}/2} C := (ud)^{N/2}
\]

\[
= C \ e^{kX} \quad \text{with} \quad k := \ln((u/d)^{\sqrt{N}/2})
\]

Approx. Sol’n:
\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
Recall: \( f(x) = (x - 1)_+ \)

\[
g(x) := f(Ce^{kx}) = (Ce^{kx} - 1)_+
\]

\[
\left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1)_+ e^{-x^2/2} \, dx \right] Ce^{kx}
\]

\[
N = 2,592,000
\]

\[
\begin{align*}
&\equiv u \\
1.00010005 \\
&\equiv d
\end{align*}
\]

\[
C := (ud)^{N/2}
\]

\[
k := \ln((u/d)^{\sqrt{N}/2})
\]

Approx. Sol’n: \[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} [g(x)] e^{-x^2/2} \, dx
\]
\[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) + e^{-x^2/2} \, dx \]
\[
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx = 1.000948567
\]

\[
\frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx
\]

\[
Ce^{ka} - 1 = 0
\]
\[
Ce^{ka} = 1
\]
\[
e^{ka} = 1/C
\]
\[
ka = \ln(1/C) = -\ln C
\]
\[
a = -(\ln C)/k
\]
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (Ce^{kx} - 1) + e^{-x^2/2} \, dx$$

\[= \frac{1}{\sqrt{2\pi}} \int_{a}^{\infty} (Ce^{kx} - 1) e^{-x^2/2} \, dx \]

\[= \frac{5000}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \]

\[a = -(\ln C)/k \]
\[
= \frac{1}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] + \sqrt{2\pi} \Phi(-a)
\]

\[
= \frac{1}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \quad \text{\[a = -(\ln C')/k\]}
\]
\[
\begin{align*}
&= \frac{1}{\sqrt{2\pi}} \left[ C \int_{a}^{\infty} e^{kx} e^{-x^2/2} \, dx - \int_{a}^{\infty} e^{-x^2/2} \, dx \right] \\
&\quad + \int_{a-k}^{\infty} e^{k(x+k)} e^{-(x+k)^2/2} \, dx \\
&\quad \underbrace{\int_{a-k}^{\infty} e^{kx} e^{k^2/2} e^{-x^2/2} e^{-k^2/2} e^{-kx}}_{\text{NEGATE THE LOWER LIMIT}} \\
&\quad \underbrace{\sqrt{2\pi} \Phi(k-a)}_{\text{DON'T FORGET}} \\
&\quad \underbrace{\int_{a-k}^{\infty} e^{-x^2/2} \, dx}_{\text{THE LOWER LIMIT}} \\
&= -\left(\ln C\right)/k
\end{align*}
\]
$$= \frac{1}{\sqrt{2\pi}} \left[ C \int_a^\infty e^{kx} e^{-x^2/2} \, dx - \int_a^\infty e^{-x^2/2} \, dx \right]$$

$$\int_a^\infty e^{k(x+k)} e^{-(x+k)^2/2} \, dx$$

$$\int_{a-k}^\infty e^{k^2} e^{-x^2/2} e^{-k^2/2}$$

$$\sqrt{2\pi} \Phi(k - a)$$

$$a = -(\ln C')/k$$
\[ a = -\left(\ln C\right)/k \]
\[
= \frac{1}{\sqrt{2\pi}} \left[ C \int_a^\infty e^{kx} e^{-x^2/2} \, dx - \int_a^\infty e^{-x^2/2} \, dx \right] \\
e^{k^2/2} \sqrt{2\pi} \Phi(k - a) - \sqrt{2\pi} \Phi(-a)
\]
\[
= \begin{bmatrix}
1.002595363 \\
C e^{k^2/2} \begin{bmatrix}
\Phi(k - a) \\
\Phi(-a)
\end{bmatrix}
\end{bmatrix}
\]
\[
= 0.024214088 \\
\begin{bmatrix}
\begin{bmatrix}
\Phi(k - a) \\
\Phi(-a)
\end{bmatrix}
\end{bmatrix}
\]
\[
a = -0.01653528434 \\
k = 0.0573390439 \\
C = 1.000948567 \\
\begin{bmatrix}
\begin{bmatrix}
\Phi(k - a) \\
\Phi(-a)
\end{bmatrix}
\end{bmatrix}
\]
\[
a = -(\ln C) / k
\]
SUMMARY:
Coin flipping problems are tractable via CLT, and useful in many applied settings, in particular, finance.

QUESTIONS?

COMMENTS?