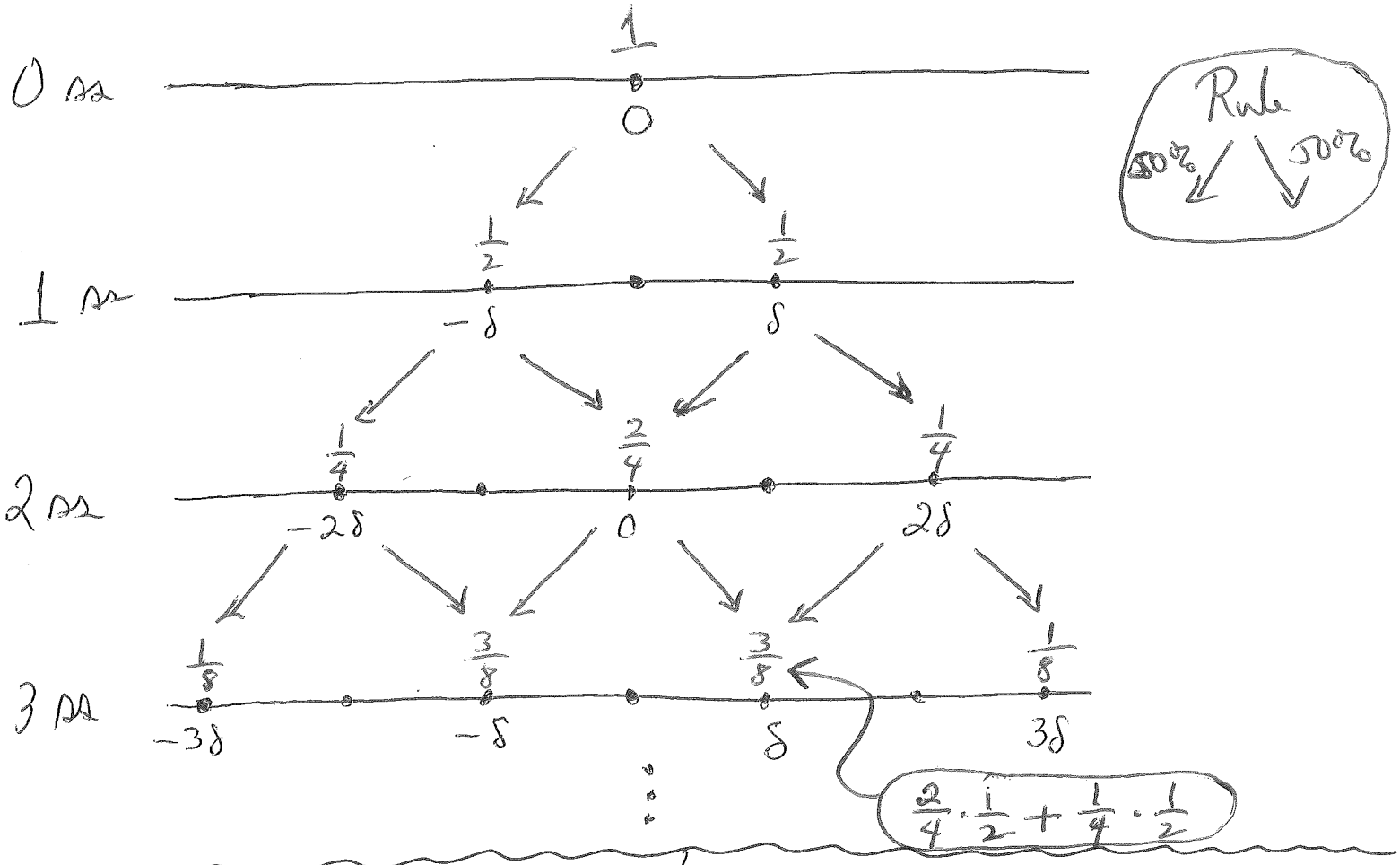
$$\boxed{N} := 10^{100} = 1 \text{ googol}$$

$$\boxed{\delta} := \frac{1}{\sqrt{N}} = \frac{1}{10^{50}} = 10^{-50}$$

$$\boxed{ss} := \text{split second} := \frac{1}{N} \text{ hrs}$$




Goal: $N\,ss = 1$ hr } Robust techniques

..., $-3\delta$ to $3\delta$ spacing $2\delta$, ..., $-N\delta$ to $N\delta$ spacing $2\delta$

## Probabilities

$$\frac{1}{1}$$

$$\frac{1}{2} \qquad \frac{1}{2}$$

$$\frac{1}{4} \qquad \frac{2}{4} \qquad \frac{1}{4}$$

$$\frac{1}{8} \qquad \frac{3}{8} \qquad \frac{3}{8} \qquad \frac{1}{8}$$

$$\vdots$$

## Denominators

$$1$$

$$2$$

$$4$$

$$8$$

$$\vdots$$

---

## Numerators (Pascal's triangle)

$$1 = \binom{0}{0}$$

$$1 = \binom{1}{0} \qquad 1 = \binom{1}{1}$$

$$1 = \binom{2}{0} \qquad 2 = \binom{2}{1} \qquad 1 = \binom{2}{2}$$

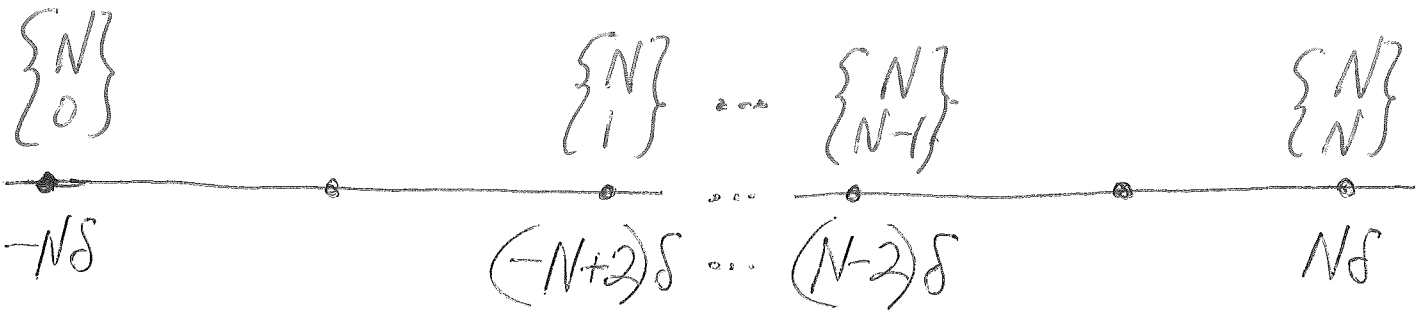
$$1 = \binom{3}{0} \qquad 3 = \binom{3}{1} \qquad 3 = \binom{3}{2} \qquad 1 = \binom{3}{3}$$

$$\vdots$$

$N_{ss} = 1$ hr

| Numerators | Denominator |
|---|---|
| $\binom{N}{0}\ \binom{N}{1} \cdots \binom{N}{N-1}\ \binom{N}{N}$ | $2^N$ |

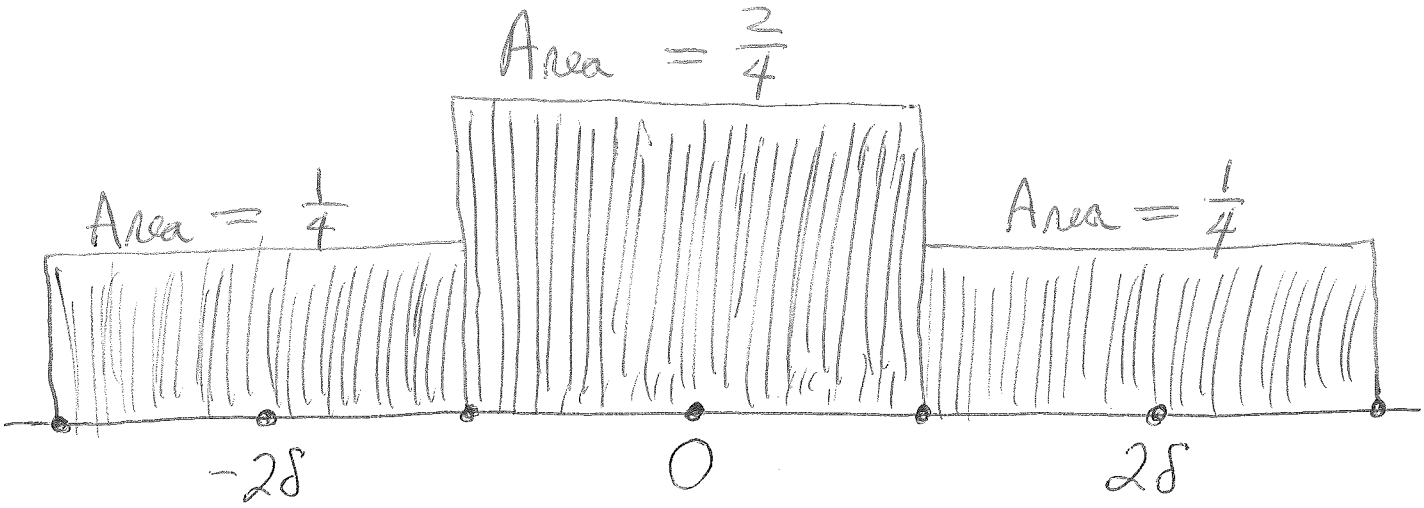
$$\forall l \in \{0,1,2,\dots,N\}, \quad \boxed{\left\{ {N \atop l} \right\}} := \frac{\binom{N}{l}}{2^N}$$




$\boxed{\text{SPACING IS } 2\delta}$

Visualize via histogram = "bar graph"

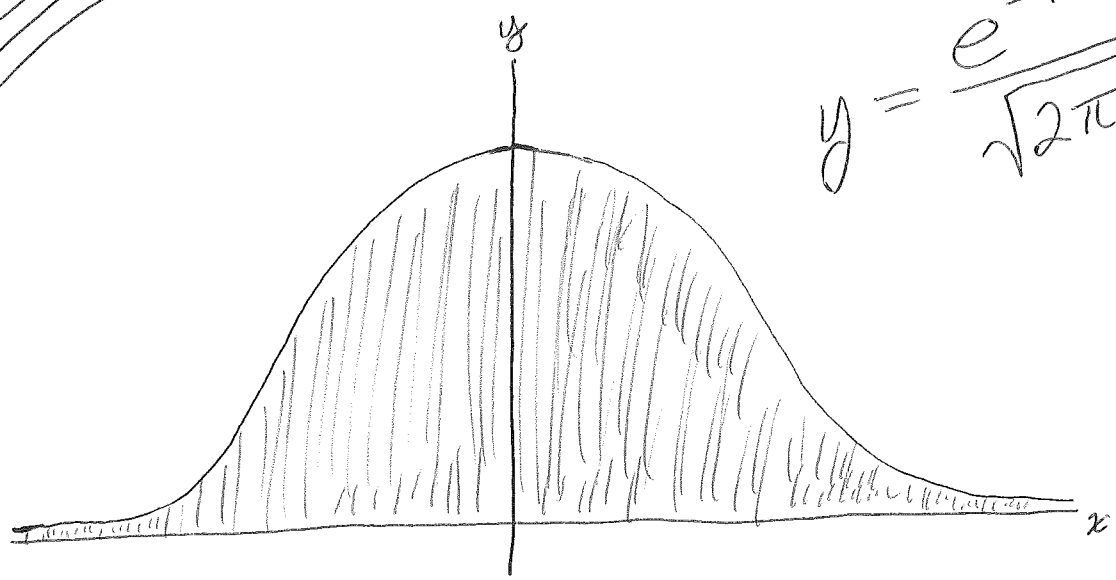
# Histogram at 2 ss




tall thin rectangles $\delta = 10^{-50}$

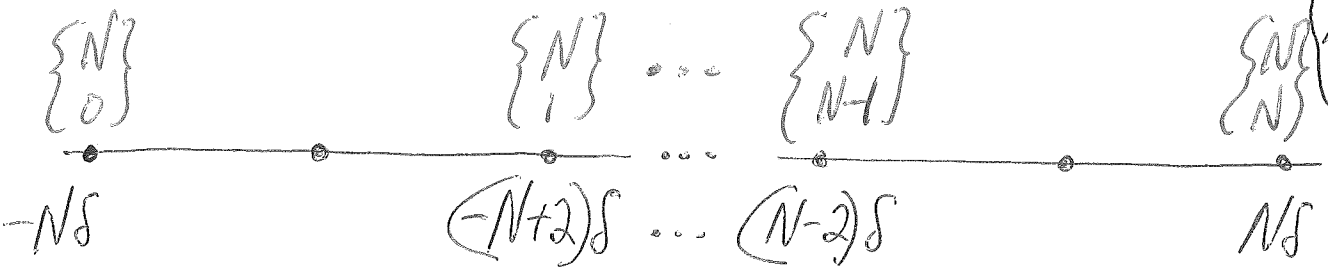
# Histogram at N ss = 1 hr?

Good

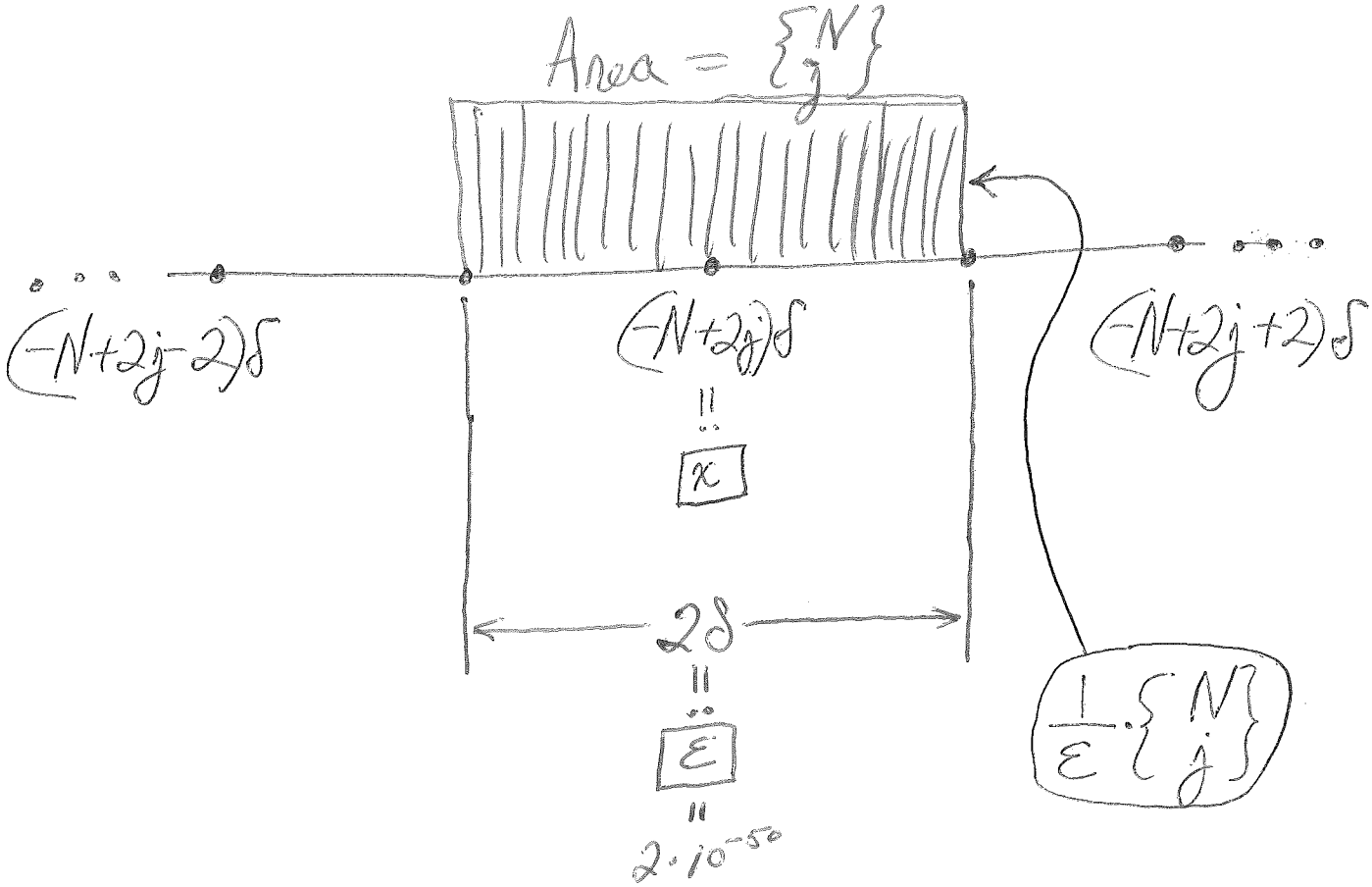



$$y = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

thin rectangles




Fix $\boxed{j} \in \{0, 1, 2, \ldots, N\}$

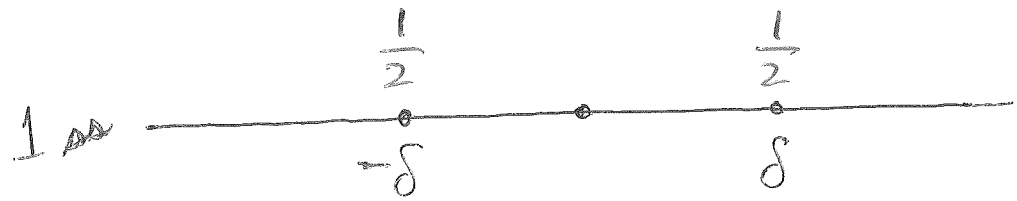



<u>Goal:</u> $\boxed{\text{LHS}} := \frac{1}{\varepsilon} \cdot \binom{N}{j} \approx \frac{e^{-x^2/2}}{\sqrt{2\pi}} =: \boxed{\text{RHS}}$

error $< 10^{-10}$
if time permits

# Fourier Transforms of Distributions

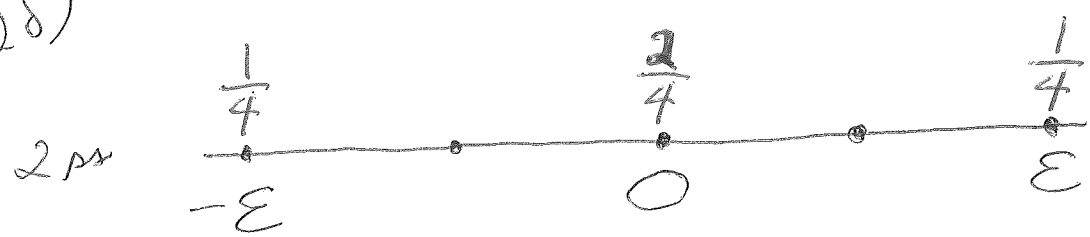





$e^{-\delta i\theta}$ $\quad$ $e^{\delta i\theta}$

$$\forall\theta\in\mathbb{R},\quad \frac{1}{2}e^{-\delta i\theta}+\frac{1}{2}e^{\delta i\theta}=\cos(\delta\theta)$$

$(\varepsilon=2\delta)$




$e^{-\varepsilon i\theta}$ $\quad$ $1$ $\quad$ $e^{\varepsilon i\theta}$

$$\forall\theta\in\mathbb{R},\quad \frac{1}{4}e^{-\varepsilon i\theta}+\frac{2}{4}+\frac{1}{4}e^{\varepsilon i\theta}$$

THE BIG IDEA

$$=\left(\frac{1}{2}e^{-\delta i\theta}+\frac{1}{2}e^{\delta i\theta}\right)^2$$

$$=\cos^2(\delta\theta)$$

$$\begin{array}{ccccc} \left\{ {N \atop j-1} \right\} & & \left\{ {N \atop j} \right\} & & \left\{ {N \atop j+1} \right\} \\ \cdots \bullet & \bullet & \bullet & \bullet & \bullet \cdots \\ (-N+2j-2)\delta & & (-N+2j)\delta & & (-N+2j+2)\delta \\ \| & & \| & & \| \\ x-\varepsilon & & x & & x+\varepsilon \\ e^{(x-\varepsilon)i\theta} & & e^{xi\theta} & & e^{(x+\varepsilon)i\theta} \\ \| & & & & \| \\ e^{xi\theta}e^{-\varepsilon i\theta} & & & & e^{xi\theta}e^{\varepsilon i\theta} \end{array}$$

$N_{DA} = 1\text{hr}$

$\forall \theta \in \mathbb{R}$

$$\cdots + \left\{ {N \atop j-1} \right\} e^{xi\theta} e^{-\varepsilon i\theta} + \left\{ {N \atop j} \right\} e^{xi\theta} + \left\{ {N \atop j+1} \right\} e^{xi\theta} e^{\varepsilon i\theta} + \cdots$$

(multiply by $e^{-xi\theta}$ & average)

(finite sum $N+1$ terms)

$$= \cos^N(\delta\theta)$$

$$\int_{-\pi/\varepsilon}^{\pi/\varepsilon} [\cos^N(\delta\theta)] \cdot e^{-xi\theta} \, d\theta$$

$$= \cdots + \left\{ {N \atop j-1} \right\} \cdot 0 + \left\{ {N \atop j} \right\} \cdot 1 + \left\{ {N \atop j+1} \right\} \cdot 0 + \cdots$$

$$= \left\{ {N \atop j} \right\}$$

(Multiply by $\frac{1}{\varepsilon}$

$$\frac{1}{\varepsilon} \cdot \left\{ {N \atop j} \right\} = LHS$$)

$$\int_{-\pi/\varepsilon}^{\pi/\varepsilon} [\cos^N(\delta\theta)] \cdot e^{-xi\theta} \, d\theta = \left\{ {N \atop j} \right\}$$

$$\underbrace{\frac{1}{\varepsilon}\cdot\fint_{-\pi/\varepsilon}^{\pi/\varepsilon}}\left[\cos^N(\delta\theta)\right]\cdot e^{-ix\theta}\,d\theta = LHS$$

$$\|$$

$$\frac{1}{\cancel{\varepsilon}_{1}}\cdot\frac{1}{2\pi/\cancel{\varepsilon}}\cdot\int_{-\pi/\varepsilon}^{\pi/\varepsilon}$$

Goal: $LHS \approx RHS$

error $< 10^{-10}$

if time permits

Goal: $\frac{1}{2\pi}\cdot\int_{-\pi/\varepsilon}^{\pi/\varepsilon}\left[\cos^N(\delta\theta)\right]\cdot e^{-ix\theta}\,d\theta \approx RHS$

error $< 10^{-10}$ if time permits

$$2\pi\cdot RHS = \sqrt{2\pi}\cdot\cancel{\sqrt{2\pi}}\cdot\frac{e^{-x^2/2}}{\cancel{\sqrt{2\pi}}_{1}} = \sqrt{2\pi}\cdot e^{-x^2/2}$$

Goal: $\int_{-\pi/\varepsilon}^{\pi/\varepsilon}\left[\cos^N(\delta\theta)\right]\cdot e^{-ix\theta}\,d\theta \approx \sqrt{2\pi}\cdot e^{-x^2/2}$

error $< 2\cdot\pi\cdot 10^{-10}$

if time permits

(heuristics, then error estimation)

<u>Goal</u>: $\int_{-\pi/\varepsilon}^{\pi/\varepsilon} \left[\cos^N(\delta\theta)\right] \cdot e^{-ix\theta}\, d\theta \approx \sqrt{2\pi} \cdot e^{-x^2/2}$

---

$$N = 10^{100} \qquad \delta = \frac{1}{\sqrt{N}} \qquad \varepsilon = 2\delta = \frac{2}{\sqrt{N}}$$

$$\frac{1}{\varepsilon} = \frac{\sqrt{N}}{2} \qquad \frac{\pi}{\varepsilon} = \frac{\pi\sqrt{N}}{2} = \frac{\pi}{2} \cdot 10^{50}$$

---

$\forall \theta \in \left[-\frac{\pi}{\varepsilon} ; \frac{\pi}{\varepsilon}\right]$,

$$\cos^N(\delta\theta) = \cos^N\left(\frac{\theta}{\sqrt{N}}\right) \approx \lim_{k \to \infty} \cos^k\left(\frac{\theta}{\sqrt{k}}\right) = e^{-\theta^2/2}$$

---

$$\int_{-\pi/\varepsilon}^{\pi/\varepsilon} \left[\cos^N(\delta\theta)\right] \cdot e^{-ix\theta}\, d\theta$$

$$\approx \int_{-\pi/\varepsilon}^{\pi/\varepsilon} e^{-\theta^2/2} \cdot e^{-ix\theta}\, d\theta$$

$$\approx \int_{-\infty}^{\infty} e^{-\theta^2/2} \cdot e^{-ix\theta}\, d\theta = \sqrt{2\pi} \cdot e^{-x^2/2}$$

(? error $< 2 \cdot \pi \cdot 10^{-10}$ ?)