
1. Find the limit, if it exists. If the limit does not exist, explain why.
   a) \( \lim_{x \to 0} \frac{e^x - \sin(x) - 1}{x^2 - x^3} \)
   b) \( \lim_{x \to 0^+} (e^x - 1)^x \)

2. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequence diverges, state whether it diverges to \( +\infty \), \(-\infty \), or neither. Show your reasoning.
   a) \( a_n = \frac{\ln(n)}{\sqrt{n}} \)
   b) \( a_n = (1 + \frac{3}{n})^{5n} \)
   c) \( a_n = (-1)^n \sqrt{n} \)

3. Let \( a_n \) be the sequence defined recursively by \( a_1 = 0, a_{n+1} = \frac{1}{4}(1 + a_n) \). Determine whether the sequence \( a_n \) converges or diverges. If \( a_n \) converges, compute its limit.

4. If \( p \) is a positive constant, show that the improper integral \( \int_1^\infty \frac{dx}{x^p} \) converges if \( p > 1 \) and diverges if \( p \leq 1 \).

5. Establish the convergence or divergence of the following series by using the comparison test.
   a) \( \sum_{n=1}^{\infty} \frac{1}{5^n - n} \)
   b) \( \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n} \)

6. Establish the convergence or divergence of the following series by using the integral test.
   a) \( \sum_{n=1}^{\infty} \frac{n^2}{e^{-n^3}} \)
   b) \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 13} \)

7. Use Pappus's theorem to find the volume of the torus (doughnut) generated by revolving a circle of radius \( r \) about a line in its plane at a distance \( R \) from its center, where \( R > r \).