1. Determine whether each of the following sequences converges or diverges. If a sequence converges, compute its limit. If a sequences diverges, state whether it diverges to $+\infty$, $-\infty$, or neither. Show your reasoning.
   a) $a_n = \frac{\ln(n)}{\sqrt{n}}$
   b) $a_n = (1 + \frac{3}{n})^{5n}$
   c) $a_n = (-1)^n \sqrt{n}$
   d) $a_n = \frac{\sin(2n)}{1 + \sqrt{n}}$
   e) $a_n = \frac{e^n - e^{-n}}{e^{-2n} - e^{2n}}$

2. Let $a_n$ be the sequence defined recursively by $a_1 = 0$, $a_{n+1} = \frac{1}{4}(1 + a_n)$. Determine whether the sequence $a_n$ converges or diverges. If $a_n$ converges, compute its limit.

3. If $p$ is a positive constant, show that the improper integral $\int_1^\infty \frac{dx}{x^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

4. Establish the convergence or divergence of the following series by using the comparision test.
   a) $\sum_{n=1}^{\infty} \frac{1}{5^n - n}$
   b) $\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^n}$

5. Establish the convergence or divergence of the following series by using the integral test.
   a) $\sum_{n=1}^{\infty} \frac{n^2}{e^{-n^3}}$
   b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 6n + 13}$

6. Find a power series representation for the following functions and determine the interval of convergence.
   a) $f(x) = \frac{1}{(1 + x)^2}$
b) \( g(x) = \frac{1}{(1+x)^3} \)

7. Show that if the vectors \( u + v \) and \( u - v \) are orthogonal, then the vectors \( u \) and \( v \) must have the same length.

8. Find an equation of the set of all points equidistant from the points \( A = (-1,5,3) \) and \( B = (6,2,-2) \). Describe the set.