7.8 (5pts)

(g) To find the radius of convergence, we check for absolute convergence, i.e. we consider the sum
\[ \sum_{n=0}^{\infty} \frac{3^n |x|^n}{2^n} = \sum_{n=0}^{\infty} \left( \frac{3|x|}{2} \right)^n . \]

This is a geometric series, which converges if and only if \( \frac{3|x|}{2} < 1 \), or in other words \( |x| < \frac{2}{3} \). Thus the radius of convergence is \( \frac{2}{3} \). To find the interval of convergence, we now check the endpoints \( -\frac{2}{3} \) and \( \frac{2}{3} \).

At \( x = \frac{2}{3} \),
\[ \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^n} \left( \frac{2}{3} \right)^n = \sum_{n=0}^{\infty} (-1)^n , \]
which diverges. At \( x = -\frac{2}{3} \),
\[ \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{2^n} \left( -\frac{2}{3} \right)^n = \sum_{n=0}^{\infty} 1 , \]
which also diverges. Thus the interval of convergence is \( ( -\frac{2}{3}, \frac{2}{3} ) \).

7.9 (1pt)

7.10 (4pts) Let \( M \) be the bound on the sequence \( \{a_n\} \), that is \( |a_n| \leq M \) for all \( n \in \mathbb{N} \). Then \( |a_n x^n| \leq M|x|^n \), so by the comparison test,
\[
\text{if } \sum_{n=0}^{\infty} M|x|^n \text{ converges, then } \sum_{n=0}^{\infty} |a_n x^n| \text{ converges.}
\]

Since the first series is a geometric series with ratio \( |x| \), this happens precisely when \( |x| < 1 \), so the power series
\[ \sum_{n=0}^{\infty} a_n x^n \text{ converges absolutely for } |x| < 1 . \]