Correction of quiz 1

**Problem 1** the plane $\mathcal{P}$ contains the points $A(-1,0,0)$, $B(0,2,-3)$, and $C(4,-5,0)$, hence it is parallel to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (for instance). The vector given by the cross product $\overrightarrow{n} = \overrightarrow{AB} \times \overrightarrow{AC}$ is thus a normal vector of the plane. A simple computation gives $\overrightarrow{n} = -15 < 1,1,1>$, and the vector $\overrightarrow{m} = <1,1,1>$ is a simple normal vector of $\mathcal{P}$.

If a point $P(x,y,z)$ belongs to $\mathcal{P}$ it has to satisfy

$$\overrightarrow{m}.\overrightarrow{AP} = 0$$

which gives

$$(x + 1) + y + z = 0.$$  

Hence a simple equation of the plane is

$$x + y + z + 1 = 0.$$  

**Problem 2** Let $\mathcal{P}$ be the plane consisting of all points that are equidistant from the points $A(2,5,5)$ and $B(-6,3,1)$. If $P(x,y,z)$ is in $\mathcal{P}$ then we have

$$|\overrightarrow{AP}| = |\overrightarrow{BP}| \iff |\overrightarrow{AP}|^2 = |\overrightarrow{BP}|^2$$

*id est*

$$(x - 2)^2 + (y - 5)^2 + (z - 5)^2 = (x + 6)^2 + (y - 3)^2 + (z - 1)^2,$$

which gives

$$4x + y + 2z - 2 = 0.$$  

**Problem 3** A normal vector of the plane $\mathcal{P}$ is $\overrightarrow{n} = <1,1,1>$. A direction vector of the line $\mathcal{L}$ is $\overrightarrow{u} = <2,-1,1>$. The dot product $\overrightarrow{n}.\overrightarrow{u} = 2 \neq 0$. Hence the line and the plane are not parallel, they have to intersect. The distance between the line and the plane is 0.