Correction of quiz 2

Problem 1 \(f(t, u, v) = tu^2e^{-tv}\). Thus:

\[
\frac{\partial f}{\partial t} = u^2e^{-tv} (1 - tv).
\]

\[
\frac{\partial f}{\partial u} = 2ute^{-tv}.
\]

\[
\frac{\partial f}{\partial v} = -t^2u^2e^{-tv}.
\]

Problem 2 The domain of \(f\) defined by \(f(x, y) = y^2\sin^2\frac{x}{x^4 + y^4}\) is \(D = \mathbb{R}^2\setminus(0, 0)\). Moreover, we have that \(f(0, y) = 0 \xrightarrow{y \to 0} 0\) but \(f(x, x) = \frac{x^2\sin^2\frac{x}{2x^4}}{2x^4} \sim \frac{x^4}{2x^4} \xrightarrow{x \to 0} \frac{1}{2}\). Hence the limit does not exist, the function is not continuous at \((0, 0)\).

Problem 3 Let \(\mathcal{L} = (0x)\) be the \(x\)-axis line, and \(\mathcal{P} = (0yz)\) the \(xy\)-plane. Let \(P \in \mathbb{R}^3\) be a point with coordinates \((x_0, y_0, z_0)\). Then

\[
\text{dist}(P, \mathcal{L}) = \sqrt{y_0^2 + z_0^2}
\]

\[
\text{dist}(P, \mathcal{P}) = |x_0| = \sqrt{x_0^2}.
\]

Thus any point \((x, y, z)\) of the surface must satisfy

\[
\sqrt{y^2 + z^2} = 2\sqrt{x}
\]

\[
y^2 + z^2 = 4x^2.
\]

This last equation defines a cone.