Correction of quiz 3

Problem 1  \[ z = f(x, y) = \ln(x - 2y). \] The domain of \( f \)
\[ D_f = \{(x, y), \ x - 2y > 0\}. \]
It is a half-plane. The point \((3, 1)\) belongs to \( D_f \) and the function \( f \) is clearly differentiable. The equation of the tangent plane of a surface \( z = f(x, y) \) at \((x_0, y_0, z_0 = f(x_0, y_0))\) is given by
\[ z - z_0 = (x - x_0) \frac{\partial f}{\partial x}(x_0, y_0) + (y - y_0) \frac{\partial f}{\partial y}(x_0, y_0). \]
In our case, \( f_x(x, y) = \frac{1}{x - 2y}, \ f_y(x, y) = \frac{-2}{x - 2y}. \) Thus an equation of the tangent plane is
\[ z = (x - 3) - 2(y + 1). \]

Problem 2  We want to know the partial derivatives with respect to \( x, y \) of the function \( P = \sqrt{u^2 + v^2 + w^2} \) where \( u = xe^y, \ v = ye^x, \) and \( w = e^{xy}, \) at \((x_0, y_0) = (0, 2)\). The chain rule gives
\[ \frac{\partial P}{\partial x} = \frac{\partial P}{\partial u} \frac{\partial u}{\partial x} + \frac{\partial P}{\partial v} \frac{\partial v}{\partial x} + \frac{\partial P}{\partial w} \frac{\partial w}{\partial x} \] (1)
and
\[ \frac{\partial P}{\partial y} = \frac{\partial P}{\partial u} \frac{\partial u}{\partial y} + \frac{\partial P}{\partial v} \frac{\partial v}{\partial y} + \frac{\partial P}{\partial w} \frac{\partial w}{\partial y}. \] (2)
We have \( u(0, 2) = 0, \ v(0, 2) = 2, \ w(0, 2) = 1. \) Thus, at \((x_0, y_0) = (0, 2), \ P = \sqrt{5}. \) Moreover, still at \((x_0, y_0) = (0, 2)\),
\[ \frac{\partial P}{\partial u} = \frac{\partial P}{\partial v} = 0, \ \frac{\partial P}{\partial v} = \frac{\partial P}{\partial u} = \frac{\partial P}{\partial w} = \frac{1}{\sqrt{5}}; \] (3)
\[ u_x = e^y = e^4, \ v_x = ye^x = 2, \ w_x = ye^{xy} = 2, \ u_y = x^2 = 0, \ v_y = e^x = 1, \ w_y = xe^{xy} = 0. \] (4)
We combine (3-4) in (1) and (2). It yields:
\[ \frac{\partial P}{\partial x}(x_0, y_0) = \frac{6}{\sqrt{5}}, \ \frac{\partial P}{\partial y}(x_0, y_0) = \frac{2}{\sqrt{5}}. \]

Problem 3  By definition,
\[ \frac{\partial f}{\partial x}(0, 0) = \lim_{x \to 0} f(x, 0) - f(0, 0) x = 0. \]
Similarly, \( \frac{\partial f}{\partial y}(0, 0) = 0. \) However, we have that \( f(x, 0) = 0 \to 0 \) whereas \( f(x, x) = \frac{1}{2} x \to \frac{1}{2}. \) Hence the function \( f \) is not continuous at \((0, 0)\).