Math 2263 - Quiz 1
Basics of geometry

Name: Score: 000000

This is a 20 minutes quiz, two-sided. No calculators, computers, cellphones, notes, book allowed. Show all work. No credit will be given for answers without work.

1. Given the vectors \( \mathbf{a} = \langle 1, -3, 4 \rangle \), \( \mathbf{b} = \langle -1, 3, 2 \rangle \) and \( \mathbf{c} = \langle 2, -6, 8 \rangle \),
   (a) (2 points) compute \( \mathbf{a} \cdot \mathbf{b} \);
   
   Solution: \( \mathbf{a} \cdot \mathbf{b} = 2 \).

   (b) (2 points) compute \( \mathbf{a} \times \mathbf{b} \);
   
   Solution:
   
   \[ \mathbf{a} \times \mathbf{b} = \det \begin{bmatrix} i & j & k \\ 1 & -3 & 4 \\ -1 & 3 & -2 \end{bmatrix} = \langle -6, -2, 0 \rangle. \]

   (c) (2 points) compute \( (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} \).
   
   Solution: Notice that \( \mathbf{c} = 2\mathbf{a} \), and we recall that the cross product \( \mathbf{a} \times \mathbf{b} \) is orthogonal to \( \mathbf{a} \) and \( \mathbf{b} \). Hence \( (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = 0 \).

2. (a) (3 points) Find a nonzero orthogonal vector to the plane \( \mathcal{P} \) through the points \( P(1, 0, 1) \), \( Q(-2, 1, 3) \), and \( R(4, 2, 5) \).
   
   Solution: The tao vectors \( \overrightarrow{PQ} = \langle -3, 1, 2 \rangle \) and \( \overrightarrow{PR} = \langle 3, 2, 4 \rangle \) are both parallel to the plane \( \mathcal{P} \), and not proportional. Hence an orthogonal vector can be given by \( \mathbf{n} = \frac{1}{9} \overrightarrow{PQ} \times \overrightarrow{PR} = \langle 0, 2, -1 \rangle \).

   (b) (3 points) Find an equation of the plane \( \mathcal{P} \).
   
   Solution: We have a normal vector to the plane \( \mathbf{n} \) and a point of the plane \( P \). Hence,
   
   \[ \mathcal{P} = \left\{ S(x, y, z) : \overrightarrow{SP} \cdot \mathbf{n} = 0 \right\} \]
   
   \[ = \left\{ (x, y, z) : < x - 1, y, z - 1 > \cdot < 0, 2, -1 >= 0 \right\} \]
   
   \[ = \left\{ (x, y, z) : 2y - z + 1 = 0 \right\}. \]

   Hence, a linear equation of the plane is given by \( 2y - z + 1 = 0 \).